Risk and Reward of Fractionally-leveraged ETFs in a Stock/Bond Portfolio

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Abstract

This article investigates 1.25X daily leveraged stock and bond exchange-traded funds (ETFs) as an alternative asset allocation for periodically rebalanced portfolios. Performance is analyzed by replicating funds from 1989-2017. Conditions for excess returns are derived analytically and confirmed empirically. Simulations are conducted to evaluate portfolio performance to provide robust assessments under a variety of market conditions. Results indicate a potential to amplify gains with a marginal reduction in Sharpe ratio. We conclude that for individual investors seeking additional returns from a stock/bond portfolio, the reduction of risk-adjusted return may be small enough to justify 1.25X leveraged ETFs over other alternatives with similar risk/reward profiles.

Keywords: Leveraged ETFs, simulation, geometric Brownian motion, bootstrapping

1. Introduction

Individual use of financial leverage is fairly common by individual investors. Many first-time home buyers use a mortgage where the buyer provides 20% of the closing price of a home, and finance the remaining 80%. The use of leverage in the stock market is also common, where most brokers offer margin accounts to support individual investors purchasing securities above their cash balance, shorting a stock, or investing in financial derivatives. These margin accounts can amplify gains and losses, but are not available in retirement fund. However, retirement accounts do not permit this use of leverage, due to the need to provide collateral and their tax exempt status[1]. Nevertheless, individual investors and financial planners seeking to boost returns can invest in leveraged funds.

Due to the effect of daily rebalancing, most leveraged exchange-traded funds (ETFs) have largely remained in the hands of short term traders. Many academic articles have discussed the technicalities behind why leveraged ETFs are not intended for passive buy-and-hold investors. Cheng and Madhaven (2009) and Guedj et al (2010) show that, due to the path dependency of leveraged ETFs, longer holding periods reduce the value of the leverage. Lu et al (2012) shows that for holding periods up to one month a 2X leveraged ETF approximately twice the return of its underlying. But, for longer holding periods, the ability of the 2X providing twice the return of its underlying diminishes. Somewhat surprisingly, some authors indicate that there are potential opportunities.

In Trainor and Carroll (2013), the term "decay" is defined when the difference between a leveraged ETF's return and the leverage factor multiplied by underlying index is negative. This article showed that low volatility coupled with a significant upward price trend can substantially offset decay. Avellaneda and Zhang (2010) show that with a dynamic hedging strategy, it is possible for active traders to manage leveraged ETFs over longer time horizons. However, the complexity of employing such an active strategy is likely beyond the skill of many retail investors. DiLellio et al (2014) find potential portfolio diversification exists with inverse and leveraged ETFs used as an alternative asset class in a long-term passive investment strategy. Diversification benefits were shown to exist, dependent on the behavior of equity and debt markets, and were shown to be generalizable to a variety of core stock and bond funds often utilized by individual investors and financial planners.

Trainor and Baryla (2008) point out that, for individual investors interested in leverage, the cost of obtaining it via a leveraged ETF can often be less than a typical margin account. Barnhorst and Copcozza (2010) discuss investors taking "volatility risk", and show how a 2X leveraged fund can under (over) perform its underlying benchmark due to higher (lower) price volatility. Giese (2010) summarizes the positive benefits of holding leveraged funds in bullish markets, but recognizes that these benefits can be offset by increased volatility.

Given the general downward trend in volatility in the equity markets since the 2008 financial crises, there may be future opportunities for individual investors to take this so-called volatility risk. As noted above, the speed at which the leveraged ETF can diverge from its underlying index is proportional to the amount of leverage. So, individual investors may have interest in using ETFs with smaller amounts of leverage than the 2X and 3X ETFs that have dominated the leveraged ETF marketplace. To this end, this article investigates whether less leverage, at 1.25X, provides a viable investment opportunity for a passively managed portfolio of stocks and bonds without taking on excessive risks. A summary of current stock and bond ETFs offering 1.25X leverage appear in Table 1 below, along with their unleveraged counterparts.

Table 1 illustrates that the assets under management for the 1.25X leveraged ETF tracking the stock and bond indices are extremely small, compared to the unleveraged ETFs with the same underlying index. Similar to other leveraged ETFs, the 1.25X leveraged ETFs also have a higher expense ratio. However, despite the 1.25X leveraged ETF's median daily share volume being hundreds of times smaller than the unleveraged version, the average spread is only ten times larger, thanks to the liquidity of S&P 500 stock components. This low volume and low spread outcome differs from what occurs for the majority of ETFs, and as shown in Agrrawal and Clark (2009), who show low volume more often leads to exponentially higher spreads. Consequently, transaction costs of rebalancing these leveraged ETFs may not significantly reduce returns, so will be investigated in more detail in this article.

The 1.25X leveraged stock and bond ETFs also exhibit larger premium/discounts. So, it is possible that during times of market stress, these ETFs may be trading in the lower or higher price range, producing a difference in holding period performance. For purposes of the analysis that follows, we assume the net effect of this premium/discount to be negligible.

Table 1 1.25X leveraged Stock and Bond ETFs and their unleveraged counterparts[2]

ETF Symbol	Underlying Index	Assets Under Management	Expense ratio	Median Daily Share Volume	Average Spread	Premium/Discount median (range)
PPLC	S&P 500	$81.11M	0.34%	25,223	0.04%	-0.03% (-2.28% to 2.25%)
IVV[3]		$144.83B	0.04%	3,870,076	0.01%	0.00% (-0.11% to 0.13%)
PPTB	Barclay's US Aggregate Bond	$25.05M	0.34%	15,100	0.04%	0.01% (-0.39% to 0.17%)
AGG		$53.62B	0.05%	3,627,118	0.01%	0.07% (-0.16% to 0.26%)

2. Data Source and Replication technique

To study long term investment performance under a variety of market conditions, we replicated returns of two equity index ETFs in Table 1 using the S&P 500 total return index (Bloomberg index code SPXT). The first fund was an unleveraged ETF, and the second was a 1.25X leveraged ETF. Since we are also interested in a mixed stock/bond portfolio, we also replicated returns for a 1.25X leveraged and unleveraged bond ETF, which are based on the Barclay's aggregate total return bond index (Bloomberg index code LBUSTRUU). We will refer to these funds simply as our leveraged or unleveraged "stock ETF" and "bond ETF" for the remainder of the paper. Since we will be replicating the effect of constant daily leverage, we obtained our stock and bond index data on a daily basis. The total return data from Bloomberg provided full years of daily returns from 1989 through 2017.

We began by replicating the unleveraged stock and bond ETF daily returns. Let r_i be the i^{th} day's total return (including dividends) of the underlying stock or bond index. To construct the unleveraged ETF, we impose an expense ratio e, so that the yearly return R can be expressed as

$$R = -1 + \prod_{i=1}^{n}\left(1 + r_i - \frac{e}{n}\right), \qquad (1)$$

where $\prod_{i=1}^{n}$ represents the product over each i^{th} day of a year with n total days. Typically, $n = 252$, corresponding to an average of 21 trading days per month.

We similarly constructed leveraged stock and bond ETF daily returns. Let $\tilde{r}_i$ be the leveraged ETF daily return (including dividends) so that, before imposing any expenses or fees,

$$\tilde{r}_i = f * r_i \qquad (2)$$

where f is the leverage factor. For a 1.25X leveraged ETF, we set $f = 1.25$. Then, the annual return $\tilde{R}$ of the leveraged ETF is

$$\tilde{R} = -1 + \prod_{i=1}^{n}\left[1 + \tilde{r}_i - \frac{\tilde{e}}{n} - \left(\frac{Libor_i + X}{n}\right)(f-1)\right], \qquad (3)$$

where $\tilde{e}$ is the expense ratio of the leveraged fund, $Libor_i$ is the annual rate for the 1-month LIBOR[4], and X is an additional financing cost imposed on the ETF provider to borrow the money and leverage the underlying index's assets. Note that, with the exception of the potentially different expense ratios, Eq. (3) reduces to (1) when $f \to 1$. Table 2 lists values that will be used for the analysis that follows. It is also important to note that no attempt to model divergence from the daily benchmark will be made, which can occur when such a fund trades at either a discount or premium. We also assume that these investments are held in a tax deferred or exempt retirement account.

Table 2 Parameters to replicate unleveraged and leveraged ETFs

Expense ratio, unleveraged fund (stock, bond)	**Leverage factor**	**Expense ratio, leveraged fund (stock, bond)**	**Additional financing cost**
e	f	$\tilde{e}$	X
0.04%, 0.05%	1.25	0.32%, 0.32%	0.25%

We can now replicate annual returns and volatility for unleveraged and leveraged stock and bond ETFs, and annual return data appears in the Appendix. The growth of a $1 investment in the 1.25X leveraged and underlying index ETF constructed with these assumptions appear in the left pane of Figure 1. The right pane of Figure 1 provides a reference to the effect that borrowing costs have on limiting return growth, where higher LIBOR rates reduced the total return for the leveraged stock and bond funds.

The *excess return*, or the positive difference between the 1.25X stock ETF and unleveraged stock ETF, grows during bull markets, then shrinks during market downturns. The longest run of growing excess returns began in 2009, helped by two conditions. First and foremost, there has not been a significant stock market downturn since 2009, and volatility has generally trended down. Additionally, the excess returns have benefited by the near 0% LIBOR rates following the 2008 financial crisis. In the next section, we derive the conditions of annual returns and volatility of the underlying ETF that produce excess returns, and compare it to these replicated returns.

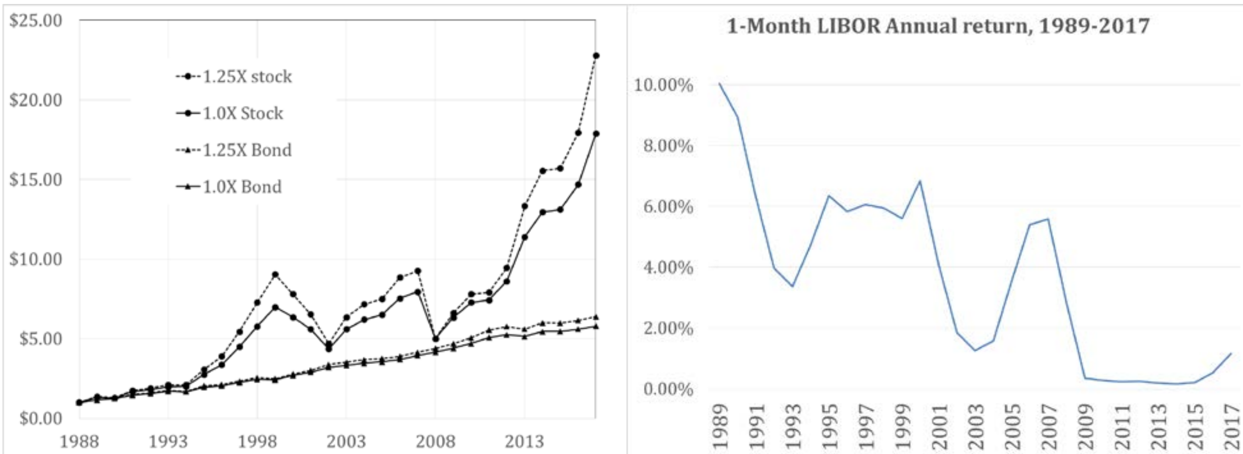

Figure 1 (left pane) 29 Years of growth of a $1 investment in replicated 1.25X leveraged and underlying stock and bond ETFs (right pane) 1-month LIBOR Annual returns over same periods

2.1 Derivation of return and volatility conditions that produce excess returns

It is well understood that excess returns of a leveraged ETF can be amplified by lower volatility, but returns on the underlying return are also important. Here, we consider the *annual* return expectations for a leveraged ETF, and compare it to the unleveraged return, net of fees and financing costs. A leveraged ETF *annual* expected returns can be expressed as

$$E(\tilde{R}) = f\,\mu - \frac{f^2}{2}\sigma^2 \qquad (4)$$

as shown in Lu et al (2012), where μ is the rate of return and σ is volatility. Thus, the ability to generate excess returns occurs when $\tilde{R} > R.$ In terms of continuously compounded expected returns, we solve

$$E(\tilde{R}) > E(R) \qquad (5)$$

for a given year. Including expenses implies that returns are reduced accordingly, so that

$$\mu \rightarrow \mu - \tilde{e} - (Libor_i + X)(f - 1) \qquad (6)$$

for the leveraged ETF and

$$\mu \rightarrow \mu - e \qquad (7)$$

for the unleveraged ETF. Then, the inequality to produce excess returns becomes

$$f\,[\mu - \tilde{e} - (Libor_i + X)(f - 1)] - \frac{f^2}{2}\sigma^2 > \mu - e - \frac{1}{2}\sigma^2. \qquad (8)$$

Solving for μ yields the quadratic relationship between volatility and returns of the underlying ETF in order to produce excess returns.

$$\mu > \frac{(f^2-1)}{2(f-1)}\sigma^2 + \frac{f[\tilde{e}+(LIbor_i+X)(f-1)]-e}{f-1}. \quad (9)$$

The first term on the right hand side of Eq. (9) contains the quadratic coefficient, and the 2nd is a constant. Since there is little correlation between the LIBOR rate and volatility[5], the average LIBOR rate from 1989-2017 of 3.57% was used that, along with the values in Table 2 for the stock ETF, simplify Eq. (9) to $\mu > 0.062 + 1.125\sigma^2$.

Figure 2 is a scatter plot of the annual returns and volatility from 1989-2017 of the unleveraged stock ETF, where the caption for each point shows the excess return. The solid line represents Eq. (9), so as expected, positive call-outs occur above the quadratic line and negative ones are below it. The data used to produce Figures 2 can be found in the Appendix.

Figure 2 Excess returns and their relationship to underlying volatility and returns, stock ETF, 1989-2017

Similar results occur for leveraged bond funds, where lower volatility and higher returns often produce higher excess returns, as shown in Figure 3. However, there is less sensitivity to higher volatility, shown by flatter quadratic. Also, for Eq. (9) to apply in Figure 3, we restricted annual returns to appear from 2009-2016, when LIBOR rates were nearly constant, and averaged 0.27%. Note that from Eq. (9) that for every 1% increase in the annual LIBOR rate, the quadratic in Figure 3 increases by 1.25%.

Figure 3 Excess returns and their relationship to underlying volatility and returns, Bond ETF, 2009-2016

2.2 Relationship of excess returns and Sharpe ratio

From the previous section, we established that for sufficiently large returns and sufficiently small volatility, excess returns are realized. In this section, we quantify when this situation has occurred from our dataset. We consider a simple linear regression of where the dependent variable is excess returns and the independent variable is the Sharpe ratio of the underlying ETF. We hypothesize that annual excess returns, $\tilde{R} - R$, *increase* with higher underlying Sharpe ratio. Formally, we express this relationship as a linear function of Sharpe ratio θ, as described in Sharpe(1994), as

$$\tilde{R} - R = \beta_0 + \beta_1\theta + \varepsilon \qquad (10)$$

where β_0 is the intercept, β_1 is the slope, and ε is a normally distributed random variable. We define the Sharpe ratio in Eq. (10) as

$$\theta = \frac{R - r_f}{\sigma}, \qquad (11)$$

where R is the annual return of the underlying ETF, r_f is the annualized risk free rate obtained from the 30-day LIBOR rate, and σ is annualized volatility. Figure 3 shows this relationship for stocks, while Figure 4 shows a similar results for bonds based on their replicated annual values from 1989-2017, where the call outs in these figures show the year.

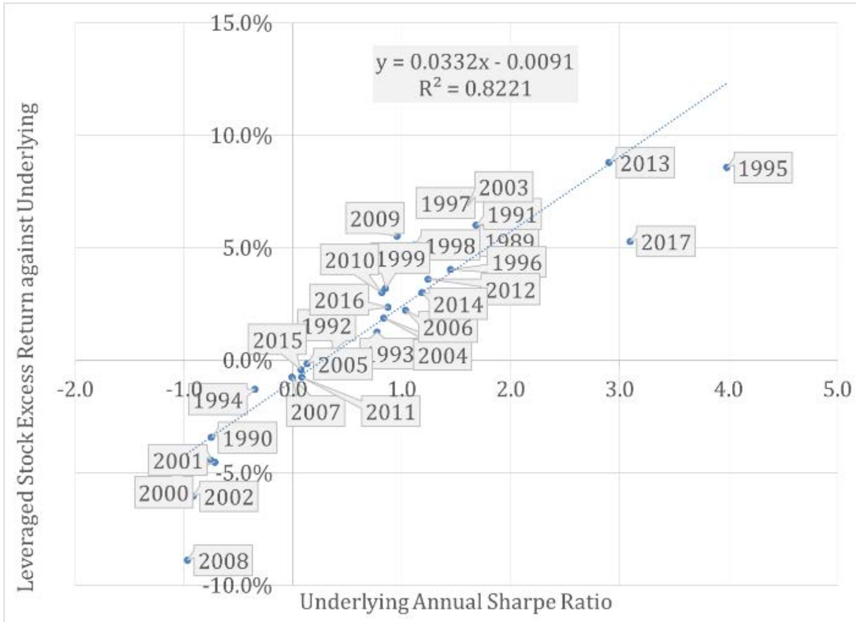

Figure 3 *Excess returns from1.25X leveraged stock ETF vs. its underlying ETF, net of fees and expenses, 1989-2017.*

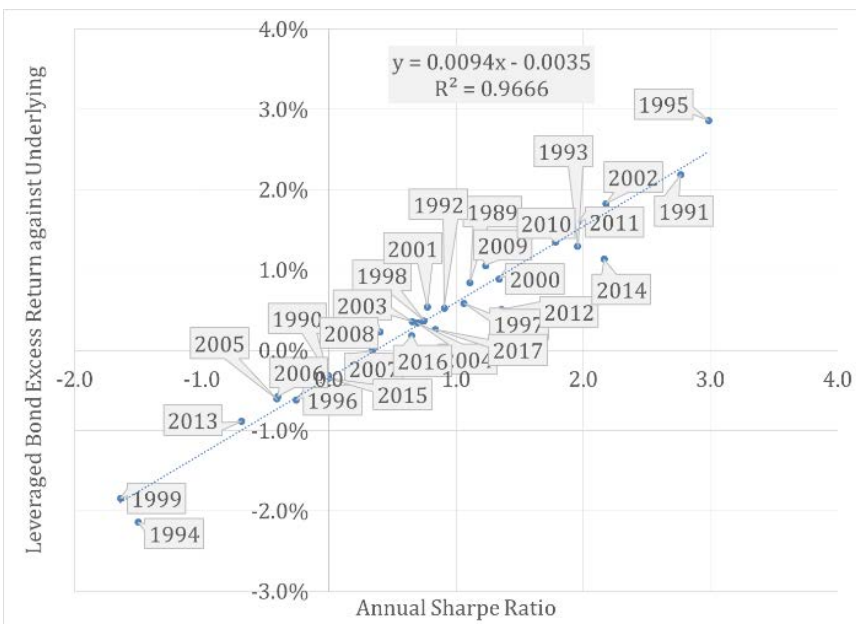

Figure 4 Excess returns from 1.25X leveraged bond ETF vs. its underlying ETF, net of fees and expenses, 1989-2017.

The values of R^2 at 82% for stocks and 97% for bonds produced in Figures 3 and 4 supports our hypothesis that the underlying Sharpe ratio can predict the degree of excess returns. The coefficients and their statistical significance appear in Table 3. All coefficients are statistically significant at either the 0.05 or 0.001 level.

Table 3 Regression coefficients and p-values for coefficients appearing in Figures 3 and 4, indicating statistical significance between the underlying Sharpe ratio and excess returns.

N=29 (1989-2017)	coefficient	p-value
β_1 (stocks)	0.0332	<0.001
β_0 (stocks)	-0.0091	<0.05
β_1 (bonds)	0.0094	<0.001
β_0 (bonds)	-0.0035	< 0.001

From these results, we can confirm that periods of upward trending prices with lower volatility will produce the highest excess returns. Conversely, periods of flat prices and high volatility will produce lower or negative excess returns. We can also predict from these regression equations that excess performance of the stock and bond ETF occur when the Sharpe ratio of the underlying exceeds 0.27 and 0.37, respectively. Lastly, we can quantify that for every unit increase in the underlying Sharpe ratio, the excess return for stocks (bonds) increases by 3.32% (0.94%).

3. Research Methods

We next investigate how a 1.25X stock and bond ETFs perform in a periodically rebalanced portfolio. Simulation methods are used to determine a distribution of 10 years of future price paths of the underlying assets. From these paths, we determine a distribution of annualized returns and Sharpe ratios for each portfolio. We selected 10,000 trials, since they produced the half-width of a 95% confidence interval that was less than 0.001[6]. In the results that follow, computational time was approximately 20 minutes on a laptop running Windows 10 with an Intel® core I7, 2.2 GHz and 6 GB of RAM. All simulation models were completed in Excel using custom VBA macros, and are available upon request.

Both of our sampling methods assume that price changes are independent of one another, so follow a Markov or memoryless process, as suggested by Fama (1965a,b). Our first approach is to simulate paths as from a geometric Brownian motion process. Our second simulation approach is often termed "bootstrapping," since it samples from a historical distribution. Two excellent references for this methodology are Davison and Hinkley (1993) and Efron and Tibshirani (1993).

3.1 Simulation of leveraged and unleveraged stock and bond prices with a GBM process

The first model is a Monte Carlo simulation that assumes prices follow geometric Brownian motion (GBM) process. Here, a future asset price at time (s_{t+1}) is found as

$$s_{t+1} = s_t e^{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma \Delta t \varepsilon}$$

where s_t is the current price, μ is the return rate, σ is the volatility or standard deviation of returns, Δt is the time increment between the current and future price, and ε is a random number obtained from the standard normal distribution.

The use of a GBM process has a long history of modeling prices of financial assets. The GBM process generates future prices that are lognormally distributed with variance growing over time, assumes returns are normally distributed, and allows the modeler to either calibrate the GBM process parameters (μ and σ) from market data or set them to some future expectation. The downside of using GBM process to model security prices is that, because it assumes returns are normally distributed, it assumes a very small likelihood of extreme (greater than a few standard deviations) returns. In practice, it is well known that "fat tails" of returns exist, meaning the probability of more extreme returns is not modeled well by the GBM process. Please see the seminal work by Fama (1965a,b) for a discussion of this behavior in commons stock. Nevertheless, modeling stock and bond ETF prices using a GBM process is fairly straightforward, and can be used to illustrate portfolio performance under a variety parameters selected.

Using a GBM process, 10-year investment returns were evaluated for a variety of rebalancing strategies. Drift and volatility were estimated from the 29 year history from 1989-2017, and appear in Table 4 for stocks, bonds and the risk-free asset. The simulation also included correlation between stock and bond returns set to -9.5%, as observed over this time period[7]. Correlations between bonds and risk-free rate and stocks and the risk-free rate were negligible over this time period, so this correlation was not included in the simulated results.

Table 4 Continuous return and volatility parameters in geometric Brownian motion (GBM) simulation

Stock		**Bond**		**Risk-free asset**	
μ	**σ**	**M**	**σ**	**μ**	**σ**
10.6%	17.8%	5.9%	3.9%	3.2%	0.2%

Additional parameters for transaction costs were also included. The bid-ask spread for stock and bonds was set to 0.01%, and for leveraged stock and bonds to 0.04%. Trading commissions were set to $4.95 per trade for an account with $200,000 at the start of each 10 year simulation.

Results for the baseline GBM simulation appear in Table 5. As expected, the mean annualized return for the unleveraged stock holdings are 9.5%, which less than the continuous return parameter of 10.6% assumed in Table 5. This effect is aptly described in Winston(2008), since over time, volatility creates a drag on the growth rate of a stock modeled by a GBM process. However, a little surprisingly, average annualized unleveraged bond returns are slightly higher. We attribute this to the much smaller volatility of the bond markets effectively eliminating volatility drag, as well as the effect of continuously vs. annually compounded rates causing a small perturbation on the numerical results[8]. Table 5 also shows the average annualized returns for the leveraged stock and bond ETFs, where excess stock annual returns are 0.8% and excess bond annual returns are 0.3%. Reviewing the mean Sharpe ratios of the stock and bond ETFs, we see that Sharpe ratios are modestly reduced by the1.25X stock and bond ETFs. We attribute this to financing costs exceeding the risk-free rate, higher expense ratio of leveraged funds, and the path dependent effect of daily leverage. The effect of leverage on Sharpe ratios is also consistent with Mulvey et al. (2007).

Table 5 also displays the performance of the 60/40 leveraged stock/bond portfolio that is periodically rebalanced. We observe that excess returns are approximately 0.8% annually, and does not appear to depend on the rebalancing interval, which is beneficial to individual investors who may prefer to rebalance once a year. Unfortunately, risk-adjusted return measured by Sharpe ratio reduced by about 5% when rebalanced annually or more. Table 5 also shows that rebalancing on a quarterly basis provided the optimal Sharpe ratio, indicating that the effect of spreads and trading commissions associated with portfolio turnover for a 1.25X leveraged stock and bond ETF do not significantly cause a drag on risk-adjusted performance.

Table 5 Averaged Annualized returns and Sharpe ratios for 10,000 trial GBM simulation

10,000 trial GBM	**Mean Annualized Return**		**Mean Sharpe Ratio**	
	unleveraged	*leveraged*	*unleveraged*	*leveraged*
100% stock ETF	9.5%	10.3%	0.118	0.114
100% bond ETF	6.0%	6.3%	0.202	0.183
60/40 stock bond portfolios				
Annual	8.5%	9.3%	0.145	0.138
Quarterly	8.5%	9.3%	0.148	0.140
Monthly	8.4%	9.2%	0.145	0.138

With our baseline results established in Table 5, we next investigated the sensitivity of these results to volatility of the stock ETF. To this end, we kept all other parameters constant, and considered high and low volatility values. As expected, decreasing volatility increases excess returns. Table 6 shows the results when stock volatility is decreased to 10% and increased to 25% with an annual rebalancing policy to 60/40 stock/bonds. Decreasing the volatility to 10% increases excess return to 1.0%, while increasing the volatility to 25% reduces excess return to 0.4%. Sharpe ratios tell a similar story, where lower volatility increases the Sharpe ratio of both the leveraged and unleveraged portfolios. However, the 5% reduction in Sharpe ratio due to leveraged shows little, if any, significant change due to changing volatility. We can conclude that the reduction in Sharpe ratio due to using the 1.25X stock and bond ETFs is insensitive to the underlying stock ETF volatility, when returns are kept constant.

Table 6 Averaged annualized returns and Sharpe ratios for above and below average long-term volatility for an annually rebalanced portfolio of 60/40 stock and bonds

Annual Rebalancing Policy Sensitivity to Volatility	**Mean Annualized Return**		**Mean Sharpe Ratio**	
	unleveraged	*leveraged*	*unleveraged*	*leveraged*
$\sigma = 17.8\%$ (baseline from Table 5a)	8.5%	9.3%	0.145	0.138
$\sigma = 10\%$ (low volatility)	8.9%	9.9%	0.260	0.247
$\sigma = 25\%$ (high volatility)	7.9%	8.3%	0.101	0.096

3.2 GBM limitations

The GBM process generally models daily returns of stocks and bonds well. However, exceptions occur. The histograms in Figure 5 shows daily returns from 1989 to 2017 for our replicated stock and bond ETFs. In the left pane of Figure 5, the normal distribution in "orange" appears to miss the peak of the observed stock market returns. Reducing σ in the normal distribution to 75% of its original value appears to improve the fit for observations near the center of the distribution, but aggravates the already poor fit of return extremes, such as the daily returns from October of 2008. The normal distribution of bond returns exhibits similar, but less pronounced differences between observed daily returns and the theoretical expectations from a normal distribution. Thus, the volatility assumption made in a GBM simulation may not fit the return distribution well. Since we know that excess returns are sensitive to volatility, an alternate simulation approach termed "bootstrapping" is used to evaluate the performance of leveraged ETFs under actual market conditions. The bootstrapping approach also benefits from imposing the risk-free rates observed during particular market periods, as opposed to modeling the evolution of risk-free asset as a GBM process.

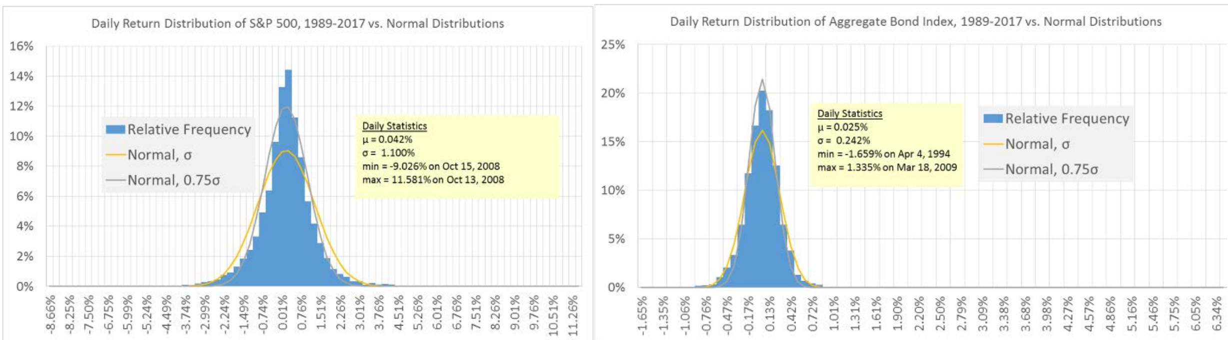

Figure 5 Daily return distribution of stock (left pane) and bond market (right pane) indices. Two normal distributions are also shown, with volatility estimates using historical returns from 1989-2017. Reducing the volatility appears to provide a slightly improved fit near the center of the distribution, but worsens the fit in the distribution tails.

3.3 Bootstrapping using 10-year historical periods

Because the return distribution shown above suffers from some modeling issues, we also investigate leveraged ETF performance using a bootstrapping model. Applying the methodology proposed in DiLellio et al (2014), we generate returns by randomly sampling empirical return histories observed from market data. By using this alternative simulation approach, the "fat tails" that occurred over the empirical return histories can be properly included in a portfolio performance evaluation, as well as the higher peaks near the center of the distribution of stock and bond returns. The following steps are adapted from the methodology found in the Appendix of DiLellio et al (2014).

Step	**Simulation Process**
1	Assign a numerical index of 1 to n for each of the daily returns observed in a historical 10-year period. Assuming 252 trading days in a year yields $n \approx 2520$.
2	For each day, determine the cumulative return up to and including the previous 21 days, so that the accumulated returns from these days are representative of the distribution of monthly returns.
3	Generate 120 random numbers ranging from 1 to n.
4	Select returns from the distribution of representative monthly returns determined in Step 2 using the random numbers found in Step 3, and generate 10 years of monthly returns. Use the same set of random numbers for each trial to select monthly returns from each of investment category (stock, bonds and risk-free asset), ensuring that the historical correlation among assets is preserved.
5	Repeat for 10,000 trials, collecting annualized return and Sharpe ratio for each trial.
6	Determine mean annualized return and Sharpe ratios found from the 10,000 trials generated in Step 5.

A key input to any bootstrapping simulation is the time period selected. To select two meaningful periods representing extremes in stock market returns and volatility, we obtained 51 years of daily returns for the S&P 500 total return index from the Center for Research in Security Prices (CRSP). We then determined 10-year simple moving average returns and volatilities, and divided them by their corresponding 51 year averages of 11.1% for annual returns and 15.1% for annual volatility, respectively. Thus, values above one represent above average returns/volatilities, and values less than one represent below average returns/volatilities. The results appear in Figure 6, where the call outs designate the final year in each 10-year moving average.

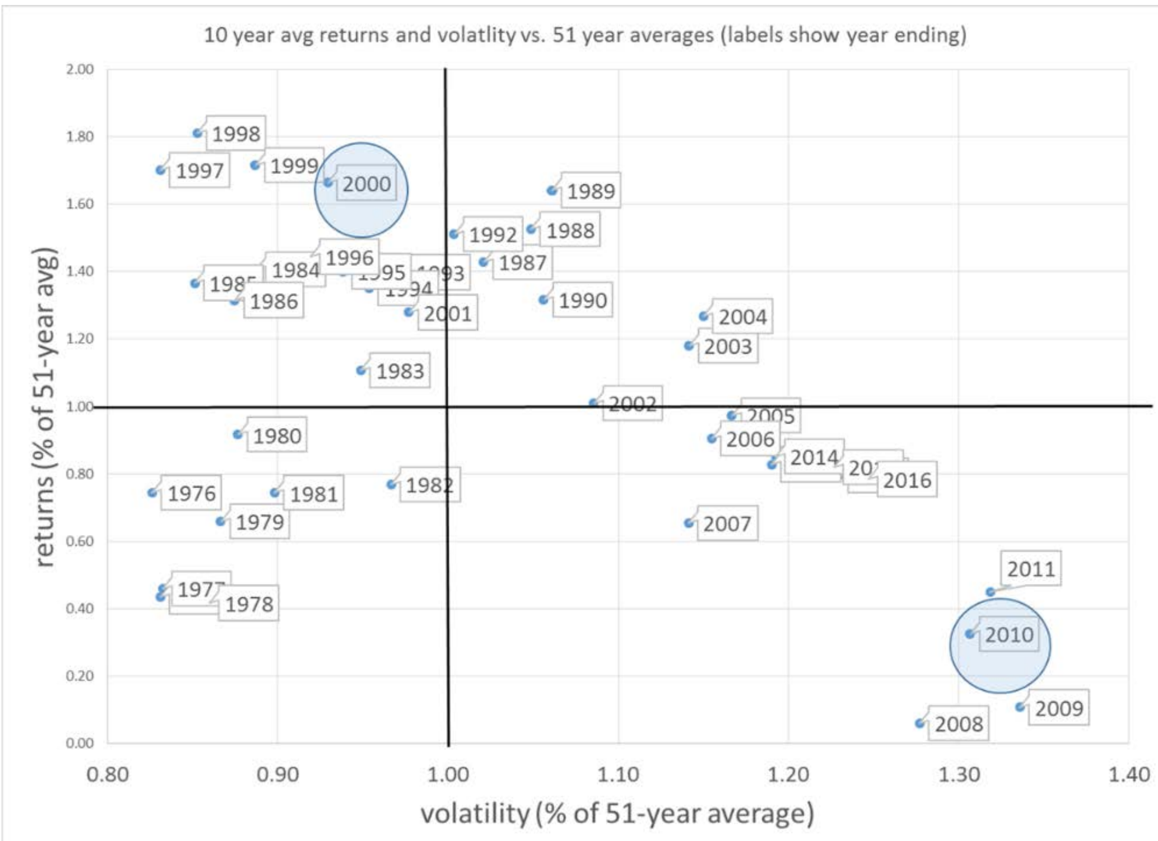

Figure 6 10-year moving average of stock market returns and volatilities divided by 51 year average. Values above one represent above average returns/volatilities, and values less than one represent below average returns/volatilities.

To select two extremes, the 10-years ending 2000 and 2010 were selected. The first period is from 1991 to 2000, when above average returns and below average volatility of stocks were observed. This first period corresponds to the equity bull market often associated with the tech bubble. The second period is from 2001-2010, when below average stock returns occurred with higher than average volatility. This second period includes the tech bubble correction as well as the financial crisis of 2008. These periods also benefit from being non-overlapping, so using them eliminates the possibility of temporal correlation.

It should be noted that there are some other interesting 10-year periods that may be worth investigating. For example, the 10 years ending in 1990 and the 1979. Unfortunately, our Bloomberg data set for stock and bond indices does not extend this far back, as noted previously. Also, the 1-month LIBOR rates were not available on a daily basis prior to the mid-80s.

Tables 7 and 8 below highlight how bootstrapping from these unique 10-year periods affect excess returns. In Table 7, the period of 1991-2000 produced excess returns from the leveraged stock(bond) ETF of 2.5% (0.3%). These returns then translate to excess returns for a 60/40 stock bond portfolio, with a value of approximately 1.7% annually when rebalanced annually, quarterly, or monthly. Thus, given the small spreads and low turnover of the 1.25X portfolio, rebalancing frequency does not affect excess returns. Again, these excess returns are due to this period containing historically lower volatilities and higher returns, as predicted by Trainor and Carrol (2013). Unfortunately, the favorable excess returns do not translate to an increase in the mean Sharpe ratio, which is similar to results from the GBM simulation approach. From Table 7, we see that the stock ETF Sharpe ratio is reduced by 3% and bond ETF Sharpe ratio is reduced by 10% with 1.25X leverage. The 60/40 stock/bond portfolios see Sharpe ratio reductions of 4%, regardless of the rebalancing frequency.

Table 7 Bootstrapping simulated results from 1991-2000, where 60/40 portfolio excess returns were approximately 1.7% annually. Sharpe ratio reduced by 4%.

10,000 trial bootstrapping 1991-2000	**Mean Annualized Return**		**Mean Sharpe Ratio**	
	unleveraged	*leveraged*	*unleveraged*	*leveraged*
100% stock ETF	17.8%	20.3%	0.269	0.260
100% bond ETF	8.0%	8.3%	0.194	0.175
60/40 stock bond portfolios				
Annual	14.0%	15.7%	0.283	0.272
Quarterly	13.9%	15.6%	0.284	0.273
Monthly	13.9%	15.6%	0.285	0.274

Table 8 shows a very different story, and summarizes the performance by sampling from the 2000-2010 return history with its high volatility and low returns of the stock ETF. Here, excess returns for the leveraged stock fund are no longer positive, with a value of -1.3%. Fortunately, the leveraged bond ETF fared better, with a 0.5% annual excess return. However, this positive excess return from the leveraged bond ETF could not be entirely offset by the leveraged stock loss, so the 1.25X leveraged 60/40 stock bond investment had negative excess returns of 0.4% annually under a variety of rebalancing policies ranging from annually to quarterly.

Results for the mean Sharpe ratio are more adversely affected when the volatility is high and returns are low. Leverage in the stock ETF reduces the averaged Sharpe ratio by 70%, while the bond ETF reduction is a more modest 8% reduction. The stock ETF Sharpe ratio reduction is extreme, since the average daily return of the leveraged stock ETF narrowly exceeded the risk-free rate in 2001-2010. Consequently, the leveraged 60/40 portfolio reduces the Sharpe ratio by 23%, regardless of rebalancing frequency.

Table 8 Bootstrapping simulated results from 2001-2010, where 60/40 portfolio excess returns were approximately -0.4% annually. Sharpe ratio reduced by 23%.

10,000 trial bootstrapping 2000-2010	**Mean Annualized Return**		**Mean Sharpe Ratio**	
	unleveraged	*leveraged*	*unleveraged*	*leveraged*
100% stock ETF	1.4%	0.1%	0.010	0.003
100% bond ETF	5.8%	6.3%	0.238	0.218
60/40 stock bond portfolios				
Annual	3.6%	3.2%	0.043	0.033
Quarterly	3.5%	3.1%	0.043	0.033
Monthly	3.5%	3.1%	0.043	0.034

4. Conclusions

This article investigated the use of 1.25X leveraged stock and bond exchange-traded funds (ETFs) as an alternative stock and bond asset allocation. Analytical conditions of return and volatility leading to excess returns over unleveraged funds is derived and agree favorably with empirical findings from 1989-2017. We also showed the relationship of excess returns and the underlying's Sharpe ratio, demonstrating high levels of statistical significance.

We used a geometric Brownian motion (GBM) process to simulate asset prices to determine a first approximation of future portfolio returns. We found that at 60/40 leveraged stock/bond fund generates excess returns of 0.8% annually (net of fees and expenses) but reduce the Sharpe ratio by 5% due to the financing exceeding the risk-free rate, higher volatility, and higher expense ratios of leveraged funds.

By using a bootstrapping simulation approach, non-overlapping periods are also evaluated using specific market conditions. We show the effect the 1.25X leverage can produce excess returns for a 60/40 portfolio of stocks and bonds of +1.7% (-0.4%) annually in higher return/lower volatility (lower return/higher volatility)) equity markets, relative to their non-leveraged counterparts, and net of fees and expenses. Again, Sharpe ratios are reduced between 4-23%, depending on the behavior of the stock and bond market.

We can conclude from these findings that a small amount of leverage, such as the 1.25X factor assumed here, can have long term net return benefits to a 60/40 stock/bond portfolio under certain conditions of the underlying asset. Additionally, while truly rationale investors may be unwilling to reduce their Sharpe ratio, the modest reduction found here may be small enough not to deter individual investors looking for slightly higher market returns without seeking the risk associated with other higher performing asset classes.

Appendix

Table A1 Annualized returns and volatility of unleveraged and leveraged stock ETF

year	n	S&P 500 index annual return	S&P 500 index Annual Volatility	S&P 500 ETF annual return	1.25X Annual Volatility	1.25X Annual return	Excess return
1989	252	31.68%	13.03%	31.57%	16.28%	36.988%	5.42%
1990	253	-3.10%	16.10%	-3.19%	20.13%	-6.54%	-3.35%
1991	253	30.47%	14.32%	30.35%	17.90%	36.47%	6.12%
1992	254	7.62%	9.73%	7.52%	12.16%	8.05%	0.53%
1993	253	10.08%	8.63%	9.98%	10.79%	11.34%	1.36%
1994	252	1.32%	9.82%	1.23%	12.28%	0.03%	-1.20%
1995	252	37.58%	7.83%	37.45%	9.78%	46.15%	8.69%
1996	254	22.96%	11.82%	22.85%	14.78%	27.00%	4.15%
1997	253	33.36%	18.16%	33.24%	22.70%	40.08%	6.83%
1998	252	28.58%	20.29%	28.46%	25.37%	33.69%	5.23%
1999	252	21.04%	18.08%	20.93%	22.60%	24.24%	3.31%
2000	252	-9.10%	22.22%	-9.19%	27.77%	-13.63%	-4.44%
2001	248	-11.89%	21.38%	-11.96%	26.72%	-16.33%	-4.36%
2002	252	-22.10%	26.04%	-22.17%	32.55%	-28.15%	-5.98%
2003	252	28.68%	17.07%	28.57%	21.34%	35.56%	6.99%
2004	252	10.88%	11.10%	10.78%	13.87%	12.75%	1.97%
2005	252	4.91%	10.29%	4.82%	12.86%	4.75%	-0.07%
2006	251	15.79%	10.02%	15.69%	12.53%	18.01%	2.32%
2007	251	5.49%	15.96%	5.40%	19.95%	4.75%	-0.65%
2008	253	-37.00%	41.05%	-37.05%	51.31%	-45.89%	-8.84%
2009	252	26.46%	27.28%	26.35%	34.09%	32.00%	5.65%
2010	252	15.06%	18.06%	14.96%	22.58%	18.08%	3.12%
2011	252	2.11%	23.29%	2.02%	29.11%	1.37%	-0.65%
2012	250	16.00%	12.70%	15.90%	15.88%	19.61%	3.71%
2013	252	32.39%	11.07%	32.27%	13.84%	41.18%	8.91%
2014	252	13.69%	11.38%	13.59%	14.23%	16.71%	3.12%
2015	252	1.38%	15.49%	1.29%	19.37%	0.95%	-0.34%
2016	252	11.96%	13.10%	11.86%	16.37%	14.32%	2.46%
2017	251	21.83%	6.67%	21.72%	8.33%	27.12%	5.40%

Table A2 Annualized returns and volatility of unleveraged and leveraged bond ETF

year	n	Aggregate Bond Index Annual Return	Aggregate Bond Index Annual Volatility	Aggregate Bond ETF Annual Return	1.25X Annual Volatility	1.25X Annual Return	Excess Return
1989	250	14.53%	3.99%	14.47%	4.98%	15.36%	0.89%
1990	253	8.96%	4.26%	8.91%	5.32%	8.64%	-0.27%
1991	253	16.00%	3.48%	15.95%	4.35%	18.18%	2.24%
1992	254	7.40%	3.73%	7.35%	4.66%	7.91%	0.57%
1993	253	9.75%	3.24%	9.69%	4.04%	11.04%	1.34%
1994	252	-2.92%	5.13%	-2.97%	6.41%	-5.07%	-2.10%
1995	252	18.47%	4.04%	18.42%	5.05%	21.33%	2.91%
1996	254	4.70%	4.62%	4.64%	5.78%	4.07%	-0.58%
1997	253	9.65%	3.34%	9.60%	4.18%	10.23%	0.63%
1998	252	8.69%	3.60%	8.63%	4.50%	9.04%	0.41%
1999	252	-0.82%	3.94%	-0.87%	4.93%	-2.68%	-1.80%
2000	252	11.63%	3.54%	11.57%	4.43%	12.51%	0.94%
2001	248	7.54%	4.38%	7.49%	5.48%	8.08%	0.59%
2002	252	10.25%	3.84%	10.20%	4.79%	12.07%	1.87%
2003	252	4.10%	4.27%	4.05%	5.33%	4.45%	0.40%
2004	252	4.34%	3.89%	4.29%	4.86%	4.67%	0.39%
2005	252	2.43%	2.89%	2.378%	3.61%	1.81%	-0.56%
2006	251	4.33%	2.78%	4.28%	3.47%	3.75%	-0.53%
2007	251	6.83%	3.50%	6.77%	4.38%	6.81%	0.04%
2008	253	5.24%	5.91%	5.19%	7.39%	5.46%	0.27%
2009	252	5.93%	4.49%	5.88%	5.61%	6.98%	1.10%
2010	252	6.92%	3.70%	6.87%	4.62%	8.26%	1.39%
2011	252	7.84%	3.85%	7.79%	4.81%	9.43%	1.64%
2012	250	3.63%	2.45%	3.57%	3.07%	4.12%	0.55%
2013	252	-2.02%	3.30%	-2.07%	4.13%	-2.92%	-0.85%
2014	252	5.97%	2.66%	5.91%	3.32%	7.09%	1.18%
2015	252	0.25%	3.77%	0.20%	4.71%	-0.11%	-0.30%
2016	252	2.65%	3.19%	2.60%	3.99%	2.82%	0.23%
2017	250	3.54%	2.80%	3.49%	3.50%	3.79%	0.30%

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Notes

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1 https://www.fool.com/knowledge-center/can-you-trade-on-margin-in-an-ira.aspx

2 Obtained from www.etf.com in January 2018.

3 IVV was selected as one of three ETFs that track the S&P 500 index, and is issued by Blackrock. The other two, VOO and SPY, issued by Vanguard and State Street Global Advisors, have similar expense ratios and spreads, and could have also been chosen. The results that follow are generalizable to using any of these S&P 500 Index ETFs.

4 https://fred.stlouisfed.org/series/USD1MTD156N

5 From 1989 to 2017, the correlation between annual volatility of the S&P500 and the annual LIBOR rate was -8.8%, suggesting no significant relationship between these two variables.

6 The primary driver on sample size was the underlying asset's volatility.

7 In the article by Johnson et al (2013), the correlation between stocks and treasury bonds is shown to change over time due to macroeconomic conditions. However, we did not attempt to include an econometric model like the one developed by these authors, instead using the long-term correlation.

8 In this example, the continuous rate for bonds was 5.9% and volatility 3.9%, so that exp(5.9% - 0.5*3.9%2) = 1.060, implying an annually compounded rate was 6%, as shown in Table 5.