# Implied Volatility Factors

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**ABSTRACT**

In the cross-section, the determinants of single stock implied volatility in the U.S. are most closely related to historical volatility, firm size, returns, and investor hedging activities. The stock options of firms in the lowest decile of our predictive model experience an implied volatility increase of 12.25% over the following month while those in the highest decile experience a decrease of 2.68%. The net differential of 14.93 volatility "points" per month is economically significant. The results are robust to out-of-sample testing and do not merely reflect a divergence between historical and implied volatility. Our analysis of the ability of implied volatility to predict future implied volatility innovations is unique as compared to prior studies of future realized volatility. A parsimonious PCA model suggests an implied volatility "capture" of 15.71% per month. Additionally, the lowest decile of portfolios sorted on predicted values of implied volatility outperforms the highest decile by 6.55% annually, indicating a positive relation between expected risk and return.

Keywords: Implied Volatility, Stock Returns, Factor Analysis, Principal Component Analysis

JEL Classification: G1 General Financial Markets, C1 Econometric and Statistical Methods and Methodolgy, C5 Econometric Modeling

## 1. Introduction

The relationships among implied volatility, realized future volatility, and future stock returns have been explored in significant detail over the past three decades. This vast literature reflects the importance of these relationships to academics and practitioners and is relevant to asset pricing, risk management, and forecasting and we apply a factor-model based approach to the issue. Large asset managers such as MSCI Barra and others employ factor models and volatility forecasting mechanisms to generate portfolio recommendations. These extensive multi-factor models first appear in academic circles in Haugen and Baker (1996) and have led to the recent rise in popularity of "smart beta" ETFs. We apply these techniques to the study of implied volatility as it relates to future changes in implied volatility and returns.

The implied volatility of options prices has been studied extensively since it provides a priced, forward-looking measure of investor expectations regarding future volatility. As might be expected, these studies find a generally positive relationship between implied volatility and future realized volatility. In early work, Latané and Rendleman (1976) demonstrate the predictive power of implied volatility for future realized volatility for twenty-four actively traded stocks. Sarwar (2005) finds that expected future volatility (proxied by implied volatility) is positively related to options trading volume in S&P 500 Index options, confirming a volume-volatility relation. Amman et. al. (2009) examine the relationships among fundamental characteristics and implied volatilities of all optionable U.S. stocks from 1996 to 2006, finding that they cannot reject the null hypothesis that implied volatility has predictive power regarding future realized volatility.[1] Christoffersen et. al. (2013) apply principal component analysis to the

[1] In contrast to these studies of single stock volatility, Canina and Figlewski (1993) show that implied volatility of the OEX (the S&P 100 Index, at the time the most actively traded options contract) is a poor predictor of future realized volatility. Jiang and Tian (2005) further demonstrate that realized volatility is a better predictor of future realized volatility in S&P 500 Index options. Similarly, Chan et. al. (2009) finds that historical volatility is not a

options of the stocks in the Dow Jones Industrial index, finding strong relationships among the factors and equity volatility, skew, and implied volatility. Buss and Vilkov (2012) use implied volatility data to construct option-implied correlations and factor betas to find a monotonically increasing risk-return relation that is not detectable with standard rolling-window betas. In a paper that is closely related to this article, DeMiguel et. al. (2013) find that the study of implied volatility can improve the selection of mean-variance portfolios with a large number of stocks as measured by Sharpe ratios. Their results are consistent with our findings that implied volatility information is useful in predicting both future volatility and returns. Various other studies provide generally consistent results for a positive relationship between implied volatility and future realized volatility, including Amman et. al. (2009), Brous et. al. (2009), Chang et. al. (2011), Dennis et. al. (2006), and Kanas (2012). The main contribution of this article is our analysis of fundamental and market information that is related to future *implied* volatility.

While the main contribution of this paper is to model fundamental and market factors in order to generate estimates of future implied volatility, we also show that expected future volatility implied in option prices is positively related to future returns. Similarly, Giot (2005) finds a positive relationship between the VIX Index and future stock returns, and we confirm this relationship using single stock data. In a recent study, An et. al. (2014) find that the top decile of firms ranked by increases in call implied volatilities outperform those in the lowest decile by approximately 1% per month, and that the return differences last for up to six months. Bali et. al. (2015) also demonstrate that expected returns are positively related to implied volatility as well

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reliable predictor of future implied volatility for S&P 500 index options, in contrast to our results. Chng and Gannon (2003) find limited information regarding future realized volatility on the Sydney Futures Exchange. Similarly, Bentes (2015) finds that GARCH forecasted volatility outperforms implied volatility in four stock markets. But these studies examine the effects of index options as opposed to a cross-section of individual stock options.

as analyst price targets. Amman et. al. (2009), Corrado and Miller, (2006), and Diavatopoulus et. al. (2008) provide additional support for this hypothesis.[2]

In this paper we also analyze the spread between realized and implied volatility as a predictor of future implied volatility. Several studies have examined this issue, and Bali and Hovakimian (2009) and Goyal and Saretto (2009) find positive relationships between volatility spreads and future stock returns. The approach of the present study is similar to these articles, but we cannot confirm that this relationship still exists in our sample from a later time period that contains the financial crisis. It may be that traders and investors used the results of the prior studies to “trade away” these potential abnormal gains, or it may be that the volatility of the financial crisis obscures the historical anomalies. However, our analysis confirms positively that a model based on predicted levels of implied volatility has the ability to forecast future economically significant, out-of-sample changes in the implied volatilities and returns of single stocks.

An efficient estimation procedure for implied volatility would be useful to academics, market practitioners and financial industry regulators. This study examines the relationships of implied volatility, realized volatility, and stock returns to various market and fundamental factors. Implied volatility is shown to be closely related to measures of historical volatility, stock trading volume, returns, and put option trading activity. Realized volatility and stock returns are also closely related to these factors. We consider forty separate factors and use the thirteen most significant factors to estimate implied volatility. On average, the stock options of firms in the lowest decile based on our predictive model experience an implied volatility increase

[2] Additional studies predict that future returns can be reliably predicted by the implied volatility skew that is observed in options prices. See, for example, Dennis et. al. (2006), Bali and Hovakimian (2009), Cremers and Weinbaum. (2010), Doran and Kreiger (2010), Xing et. al. (2010), Bali and Murray (2013), and Le and Zurbruegg (2014).

of 12.25 percent over the next month while those in the highest implied volatility decile experience a decrease of 2.68 percent, for a net differential of 14.93 volatility "points" that is economically significant. The results are robust to out of sample testing and do not merely reflect a divergence between historical and implied volatility. Further, Principal Component Analysis identifies four highly economically important factors in the implied volatility generating process – "realized volatility," "size," "returns," and "hedging activity." A parsimonious model with four factors constructed from seven variables suggests an implied volatility "capture" of 15.71 percent per month. Finally, we demonstrate a positive relation between volatility expectations (risk) and return, as the lowest decile of portfolios sorted on predicted values of implied volatility outperforms the highest decile by about 6.55 percent annually using two separate estimation procedures.

## 2. Empirical Analysis

### 2.1 Data

We collect individual monthly stock information from three separate databases for the period from January 1, 2005 to June 30, 2013. We utilize implied volatility data for all stocks in the U.S. that are available on Bloomberg Professional® and we collect three month "at the money" (hereafter ATM) implied volatility levels, which Bloomberg calculates as an average of ATM call and put volatilities over an interpolated constant maturity. We gather fundamental individual stock information from Compustat and stock price data from CRSP. There are 2,867 individual stocks and 102 months in our initial sample, providing a total of 292,434 firm-month observations. Where there are obvious outliers, the data is winsorized at the one percent level at both extremes. Summary statistics for the data set can be found in Table 1 and a correlation matrix of these variables is presented in Table II. Since the focus of the study is implied volatility, in both of these tables the variables are sorted in order of highest absolute correlation with three month implied volatility. The previously documented close relationship between historical and implied volatility is reflected at the top of the table. Implied volatility is also closely related to firm size (lnmktcap), stock dollar turnover (dolturn), dividend yield (divyldw), and a host of other variables. Complete details regarding the definition and construction of the explanatory variables are contained in the Appendix.

## 2.2. Implied Volatility Modeling

In order to examine the relationships among implied volatility and the explanatory variables, we implement a variation of the Fama-MacBeth (1973) procedure. Instead of examining stock returns as the dependent variable as in the prior literature, we study implied volatility as the dependent variable of interest. The approach is analogous to Haugen and Baker (1996, 2008) who examine a variety of fundamental, risk, liquidity, and other factors to explain stock returns. It is also quite similar to factor-based return and volatility forecasting that has been utilized for decades as a tool in portfolio construction, most notably by MSCI Barra. For a detailed exposition of this approach, the reader may refer to Grinold and Kahn (2000).

For each month in our sample, we estimate the cross-sectional relationships among the explanatory variables and implied volatility using an ordinary least squares (OLS), cross-sectional, multiple regression analysis. Haugen and Baker (1996 & 2010) choose their variables based on t-statistics from univariate models to implement a “stepwise” regression procedure. In substantially the same fashion, we choose variables based on their absolute correlation with three month implied volatility. We begin by estimating a full sample model that uses 90-day historical volatility (that has the highest correlation with three month implied volatility) as the only independent variable. We then add the explanatory variables from Table 2 to the regressions and retain them if they are significant at the ten percent level and do not diminish the statistical significance of the prior accepted significant variable. If a particular variable does reduce the significance of the prior variable to less than ten percent, we run estimations based on the other accepted independent variables and the next variable.. We retain the variable that provides the highest monthly cross-sectional average R-squared value and continue to add additional variables. Following this procedure we find the following thirteen variables provide the greatest explanatory power for three month implied volatility:

- 90-, 180-, and 30-day historical stock volatility
- market capitalization (abbreviation)
- dividend yield
- net profit margin
- the ratio of shares traded over shares outstanding
- total dollar stock turnover
- analyst estimates of the firm's five-year growth rate
- one month excess returns
- total options volume
- the ratio of put options traded to put option open interest
- the ratio of all options traded to option open interest

Based on this analysis, we generate three-month implied volatility estimates for the monthly cross-section of stock options as a function of these thirteen variables. Specifically, for each stock in our sample we estimate the following equation with standard errors corrected for heteroskedasticity by the Newey and West (1987) procedure with three lags (one quarter of data):

$$IV_{j,t} = \sum_{i=1}^{13} \hat{P}_{i,t} * F_{j,i,t} + u_{j,t},$$

where

$IV_{j,t}$= three-month ATM implied volatility for each stock *j* in month *t*,

$\hat{P}_{i,t}$= regression coefficient or payoff to factor *i* in month *t*,

$F_{j,i,t}$= exposure (firm characteristics such as historical volatility, size, dividend yield, profitability, etc.) to factor *i* for stock *j* that is observable at the end of month *t*.

$u_{j,t}$= the unexplained component of implied volatility for stock *j* in month *t*.

We then compute the average of the monthly cross-sectional coefficients and use these monthly average coefficient values to estimate predicted values of implied volatility for each of the firm-months in our sample. We only use variables that are observable at time *t* to create estimated values that we then compare to actual levels of implied volatility at the same month-end time *t*. Thus, for example, we utilize the figures for the most recently reported quarterly value of net profit margin (and the other variables that are available on a quarterly or monthly basis) to estimate predicted implied volatility levels for a given month-end. To illustrate, for the first month of our sample, there is only one month of cross-sectional results to report, and we examine the following one-month change in implied volatility and returns. For each *n*th month, there is an *n*-month average of coefficients that is used to estimate implied volatility levels for the next month. The average monthly R-squared value for the concurrent estimations is 72.4 percent, and we provide a graph of the monthly R-squared values for the entire sample period in Figure 1. In the early years of the study the R-squared values fall steadily from over 0.90 to hover between 0.60 and 0.80 for a few years, and then fall into a lower band since the global financial crisis that ends in 2010. This is natural since we are including more months in each subsequent sample that encompass successively more months (and greater variability), and it may also be true that traders and investors become aware of these factors over time and during the volatility of the financial crisis, making the model less reliable as market participants act on this type of information. An alternative explanation is that the general uncertainty created during the financial crisis leads to a decline in the amount of information provided by previously important and relevant factors in the implied volatility generating process.

The results of the average coefficient estimations are contained in Table 3. In the first column we present the coefficients for a model based on our full sample period from January 2005 to June 2013. The highest absolute t-statistics appear towards the top of the table and decline down the column. All of the variables are significant at the one percent level with the exceptions of net profit margin and the total option turnover ratio that are significant at the ten percent level. However, as demonstrated in Table 3, implied volatility is clearly related to a variety of fundamental factors in addition to historical realized volatility. It is generally negatively related to firm size, dividend yield, and net profit margin, three measures of company stability. Smaller, less profitable firms that pay smaller dividends might reasonably be expected to experience higher stock volatility. Regarding trading in company shares, implied volatility is also negatively related to the dollar amount of stock traded (which may again be related to size), but positively related to the stock turnover ratio, the amount of shares traded divided by total shares outstanding. It may be that a higher ratio of shares being traded results in additional volatility that is reflected in implied volatility expectations. Analyst expectations regarding firms' long- term growth rates are positively related to implied volatility, a result that may be viewed as the inverse of the stability variables as the shares of higher growth firms are generally more volatile. One month excess returns are negatively related to implied volatility which is evidence of the asymmetric volatility phenomenon as volatility rises during periods of negative returns. Total options volume and the total options turnover ratio are positively related to implied volatility. The first relationship may be a result of options traders' preference for high implied volatility stocks, while the second one may again reflect higher levels of volatility information being impounded into options prices (paralleling the result for the stock turnover ratio). Finally, the put turnover ratio is negatively related to implied volatility, which seems counter-intuitive since a high ratio of puts traded to open interest in put options should seemingly drive implied volatility higher if put buyers are purchasing insurance against price declines. But it may be the case that and informed put buyers are undertaking hedging activities in stocks that subsequently perform well, resulting in lower levels of implied volatility (the inverse of the "asymmetric volatility phenomenon"). Or it may be the case that put buyers purchase "out-of-the-money" options that do not greatly affect our proxy for implied volatility that is ATM. Such purchases may underlie the results of the previously cited papers regarding single stock volatility skews. We explore these results in further detail in Section 3 in our discussion of Principal Component Analysis.

In the second column of Table 3 we re-estimate the model using only the first 89 months in our sample to May 2012 in order to forecast implied volatility over the following year. The coefficients, t-statistics, and R-squared estimates are quite similar to those of the "full" model, thus the results are robust to out-of-sample testing. For both models, we calculate "percentage residuals" by dividing the residual model value (predicted minus actual) for each month by the current level of implied volatility (the dependent variable). We do so because we want to forecast percentage changes in implied volatility, not changes that could be biased by the overall level of implied volatility. Using these percentage residuals, we form deciles for each month of the sample and calculate mean implied values. The stocks in decile 1 have the highest percentage residual values (implied volatility is most "undervalued" compared to the predicted values) while the stocks in decile 10 have the lowest percentage residual values. The results of this process are presented in Table 4. Panel A provides the mean implied volatility values for the both sample periods sorted by these deciles, showing that the average implied volatility of the lowest decile stocks is 67.22 percent and the average implied volatility of the highest decile is 37.38 percent. This table indicates the even though the mean implied volatilities in Decile 1 are highest, they are also the ones that show the largest underestimates of volatility as estimated by our models, both in the full sample and in the estimation to May 2012.

The full implications of these models are contained in Panel B of Table 4, which presents the next month change in implied volatility across the deciles. For the full estimation model, next month implied volatility rises by 12.25 volatility "points" for the first decile, while next month volatility falls by 2.68 points for the most "overvalued" decile 10. We use actual changes in volatility instead of percentages here because an actual trading strategy based on these models would go long/short equal amounts of "vega" and could capture the full amount of the difference in the changes in volatilities. The difference between decile 1 and decile 10 is 14.94 volatility points that is an economically significant result, since trading desks at hedge funds and banks routinely hold positions in excess of fifty thousand vega in single stocks, and our results are generated on a monthly basis that would be replicated twelve times per year. The t-statistic for the pairwise comparison of means is highly significant at 19.79. In the interest of robustness, we also use the model results up until May 2012 to estimate out of sample predictions from June 2012 to June 2013, and the results are even better. There is a 21.19 monthly volatility point differential between decile 1 and decile 10. These results are generated using the model estimate up until May 2012 without updating thereafter. A further (unreported) model that is updated for each month following May 2012 generates a volatility point differential of 19.28 percent.

Finally, in Panel C of Table 4, we sort next month returns by the predicted implied volatility deciles and stocks in the lowest decile outperform those in the highest decile by 0.53 percent monthly, or 6.55 percent annually. Thus the stocks experience the highest increases in implied volatility also experience higher returns, confirming a positive relationship between risk and return. This result may provide further information to investors and traders as they balance the risk/return tradeoff, and this information may prove highly useful to options market makers as they set bid-ask spreads and hedge single stock options positions.

While these results are highly significant in economic terms, the application of a strategy to capitalize on them is most probably limited to large options market makers. In each of the months in our out of sample forecasts, deciles one and ten contain an average of 239 single stocks, with a range between 232 and 243. Similarly, Bali and Hovakimian (2009) report their results for quintiles over their sample from 1996 to 2005, and it would be necessary to trade in approximately 365 transactions per month on each side (long/short) of their proposed trading strategy. Similar results obtain for the study of Goyal and Saretto (2009). Only large option market makers are equipped to handle these types of positions and they have the added advantage of receiving, rather than paying the bid-ask spread. Additionally, all of these positions would be subject to "slippage" in the form of delta-hedging each single stock position on a daily basis (paying the bid-ask spread on stocks) as well as potential stock commissions. These costs could be reduced through the use of single stock variance swaps, but strategies that capitalize on our results would most likely only be employed by the most sophisticated options traders. Nevertheless, our results shed light on the potential drivers of single stock implied volatility as well as the relationships among implied volatilities and realized volatility that we explore in the next section.

### 2.3. Modeling the Spread between Historical and Implied Volatility

We have established that there is a relationship between certain fundamental and technical stock factors and implied volatility and that a model based on these factors is a reliable predictor of future changes in implied volatility for U.S. equity options. In this section we further explore how this relationship is related to the spread between historical three month volatility and three month implied volatility based on the previously reported results of Bali and Hovakimian (2009) and Goyal and Saretto (2009). In Table 5 we present the results of estimations using the same independent variables that we use in Table 3. However, for these estimations we use the spread between three month implied volatility and three month realized stock volatility as the dependent variable, as in Bali and Hovakimian (2009) and Goyal and Saretto (2009). In both the full sample period and in the subsample, the regression coefficients and t-statistics are quite similar to those of Table 3, although the average R-squared values fall. There is one important exception, however. The coefficient for 30-day historical volatility switches from 0.16 to -0.83 in both the full model and the estimation to May 2012, and remains significant at the one percent level. This negative coefficient is indicative of the process by which the spread between our predicted values of volatility and implied volatility is attenuated. When the estimates provided by our model reach extreme values, thirty day realized volatility brings the relationship back into line (i.e. the difference between observed implied volatility and that predicted by the model is reduced).

This result is explored further in Table 6, where Panel A provides the mean differences between three month implied and 90-day historical volatility for each of the deciles estimated by the model. As expected, the difference is negative (-21.21 volatility points) for decile one where implied volatility is lower than its historical performance, and higher (10.58) in decile ten. The results are consistent in both the ex post results as well as the out of sample period without updating of the model. However, in Panel B the next month differences are much smaller, as the next month 30 day realized volatility acts to shrink the spread. This is the result predicted by the estimations in Table 5, and the spread rises from -21.21 to -5.37 for decile one and falls from 10.58 to 4.58 for decile ten in the full estimation period. Similar results obtain for the out of sample results, and Panel C shows the overall changes in the implied/realized volatility spreads. On average, the monthly difference in spreads between deciles one and ten shrink 21.83 (21.88) volatility points for the full model (out of sample estimation), demonstrating the ability of implied volatility spreads as estimated by our model to forecast future one month realized volatility. However, in Panel D, we observe that portfolio deciles based on the changes in the implied/realized volatility spread do not have predictive ability for next month returns. This result differs from the results of Bali and Hovakimian (2009) and Goyal and Saretto (2009), who find a positive relationship between volatility spreads and future stock returns from 1996 to 2005 and 2006, respectively. Their results suggest that perhaps the return differences are driven solely by the difference in realized and implied volatility so it may not be necessary to analyze the fundamental and technical factors we include in our estimations. Thus we explore the execution of a trading strategy that sorts our sample into deciles based on the difference between implied and historical volatility. Panel A of Table 7 presents just such a procedure and we observe that the differences between implied and historical volatility are even greater than those based on our estimations. Parallel to the results of Table 6, Panels B and C, these differences are attenuated over the next month as the differences in spreads attenuate by 33.29 volatility points for the full model and 36.37 in the out of sample period, which are even larger than the differences in the deciles sorted by the divergence from our regression estimates. However, when we look at next month changes in implied volatility in Panel D, we observe that the mean increase in implied volatility for decile one is only 2.00 percent and it is not statistically different from zero. The change for decile ten is statistically significant at 9.37 percent, but much lower than the increase of 15.84 percent we observed for decile one in Panel C of Table 6. Additionally, this strategy suggests the purchase of volatility of the firms where implied volatility is actually *higher* than its historical levels. And without a matching basket of short positions (since the change for decile one is not significant), this is not a particularly appealing strategy in terms of intuition or potential risk.

## 3. Principal Component Analysis

While we have established that our models generate economically significant predictions of changes in implied volatility, we also seek to understand the economic factors driving these fluctuations. Thus we conduct Principal Component Analysis (PCA) to extract potential factors that explain the variance of our explanatory variables and relate those factors to changes in implied volatility.

The first step in our analysis is to include all thirteen of our explanatory variables in a PCA model, and the initial results are presented in Table 8. These results suggest a total of eight factors may be present according to Kaiser's rule (eivgenvalue greater than or equal to one). A scree plot (not shown) also shows a sharp drop following component eight. The cumulative variance explained by the first eight components totals 85.6 percent. However, an examination of the rotated (orthogonalized) factor pattern reveals that some of the components include only one variable. Since PCA is essentially a variable reduction procedure, we begin reducing the number of components from "right to left" in Table 9 by removing those variables that "load" on just one component at the 100 percent level. After each removal we examine the eigenvalues of the new PCA rotation to make sure the percent of variance explained remains above 80 percent.

The results of this process are contained in Table 10 where we present a revised factor analysis that uses four principal components and seven variables. We denote the first component as "realized volatility" since the three historical volatility variables load on that component. The second component is denoted as "size" since it includes stock turnover and dollar volume as significant loadings. The third and fourth components are denoted as "returns" and "hedging activity" since one-month excess returns and put trading turnover load on these components, respectively. In the interest of brevity we do not include an updated table of eigenvalues, but the cumulative percentage of the variance explained by this model is 88.7 percent, which is actually higher than the 85.6 percent observed in the original model.

Now that we have identified some economically meaningful factors in our sample of explanatory variables, we seek to examine how these principal components are related to implied volatility. We estimate Newey and West (1987) regressions using implied volatility as the dependent variable and predicted values of the principal components as the explanatory variables. The results of these estimations are contained in Table 11 and are economically meaningful. The coefficient for "realized volatility" is positive and strongly significant (t-stat = 65.69), as would be expected. The remaining coefficients are all negative and significant at the one percent level. The negative coefficient for "size" reflects the higher volatility of smaller firms versus larger ones. The negative coefficient for "returns" reinforces the earlier finding that asymmetric volatility remains present in U.S. stock returns. And once again, implied volatility is negatively related to put trading and is likely to be related to hedging activities in single stocks that is largely conducted through OTM put options.[3]

The final step in our analysis is to examine whether predictions based on these regressions provide economically different outcomes for stock implied volatilities sorted by deciles of predicted values. We once again sort the predicted values of implied volatility into deciles and compare next-month changes in implied volatility. The results of these sorting procedures are presented in Table 12. Panel A, which is analogous to Panel B of Table 4, shows that the implied volatility differential between the lowest and highest deciles is 15.71 percent, a slight improvement over our initial Fama-MacBeth (1973) sorting result of 14.94 percent. Return deciles are presented in Panel B, the result is an almost identical 6.51 percent annual return (rounded 0.53 monthly) differential compared to the 6.55 percent result (0.53 rounded monthly) obtained in Panel C of Table 4. Thus even though the contemporaneous return-volatility relationship is negative as shown in Section 2.2 (the "asymmetric volatility phenomenon"), expectations regarding volatility (risk) are positively related to future returns, which is consistent with financial theory.

## 4. Conclusion

This study provides evidence that implied volatility is closely related to various market, fundamental, and technical factors. Implied volatility is shown to be closely related to measures of historical volatility, stock trading volume, returns, and put option trading activity. On average, the stock options of firms in the lowest decile based on our predictive model experience

[3] Blau and Wade (2013) find that short selling activity dominates put-buying activity as a predictor of future stock returns. Additional studies (e.g. Chan et. al. (1993), Easley et. al. (1998), and Stephan and Whaley (1990)) provide inconclusive results on this issue.

an implied volatility increase of 12.25 percent over the next month while those in the highest implied volatility decile experience a decrease of 2.68 percent, for a net differential of 14.93 volatility "points" per month that is economically significant. The results are robust to out of sample testing and do not merely reflect a divergence between historical and implied volatility. Further, Principal Component Analysis identifies four economically important factors in the implied volatility generating process – "realized volatility," "size," "returns," and "hedging activity." A parsimonious model that includes only four factors constructed from seven variables suggests an implied volatility "capture" of 15.71 percent per month. Finally, we demonstrate a positive relation between volatility expectations (risk) and return, as the lowest decile of portfolios sorted on predicted values of implied volatility outperforms the highest decile by about 6.55 percent annually using two separate estimation procedures.

**Appendix**

Variable Definitions

The following list provides describes the variables collected and calculated for the study that are first presented in Table 1. They are listed in order of the absolute value of their Spearman cross-sectional correlation with three month ATM implied volatility.

| | |
|---|---|
| iv3m: | Three month implied volatility. |
| hv90: | Historical 90-day stock return volatility. |
| hv180: | Historical 180-day stock return volatility. |
| hv30: | Historical 30-day stock return volatility. |
| iv3ml~12: | 12 month lag of 3 month IV. |
| lnmktcap: | The natural logarithm of market capitalization. |
| turn_bb: | Stock volume/shares outstanding. |
| divyldw: | Dividend yield. |
| beta: | Stock beta relative to S&P 500. |
| npm_obs: | Net profit margin observable at time t-1. |
| peg: | Price/Earnings to growth ratio. |
| $turn: | Dollar Stock Turnover. |
| roa: | Return on Assets. |
| roe: | Return on Equity. |
| ltg: | Long-term growth rate (analyst's estimates). |
| ltd_eq: | Long Term Debt/Equity Ratio. |
| totdbt~q: | Total Debt. |
| resid: | Residuals from equation estimating "Abnormal Accruals." |
| xsret12: | Twelve month Excess Returns. |
| p_fcf_bb: | Price to Cashflow ratio from Bloomberg. |
| mktbook | Market to Book Value Ratio. |
| vol_put: | Put option trading volume. |
| tang: | "Tangibility," the ratio of tangible assets to total assets. |
| volume: | Stock share trading volume. |
| p_sale~b: | Ratio of Price/Sales. |
| vol_cal: | Call option trading volume. |
| ltg_sd: | The standard deviation of analyst's estimates of long-term growth (ltg). |
| put_oi: | Put option open interest. |
| xsret1: | One month excess returns. |
| os: | The ratio of options to stock trading volumes. |
| tot_oi~K: | Total options open interest (000's). |
| pe_bb: | Price/Earnings ratio. |
| tot_op~K: | Total options volume (000's). |
| cal_oi: | Total call options volume (000's). |
| put_turn: | Put options turnover (put option volume/open interest). |
| cal_turn: | Call options turnover (call option volume/open interest). |
| eps_sd~t: | Standard deviation of analyst earnings estimates. |
| opt_turn: | Total options turnover (total option volume/open interest). |

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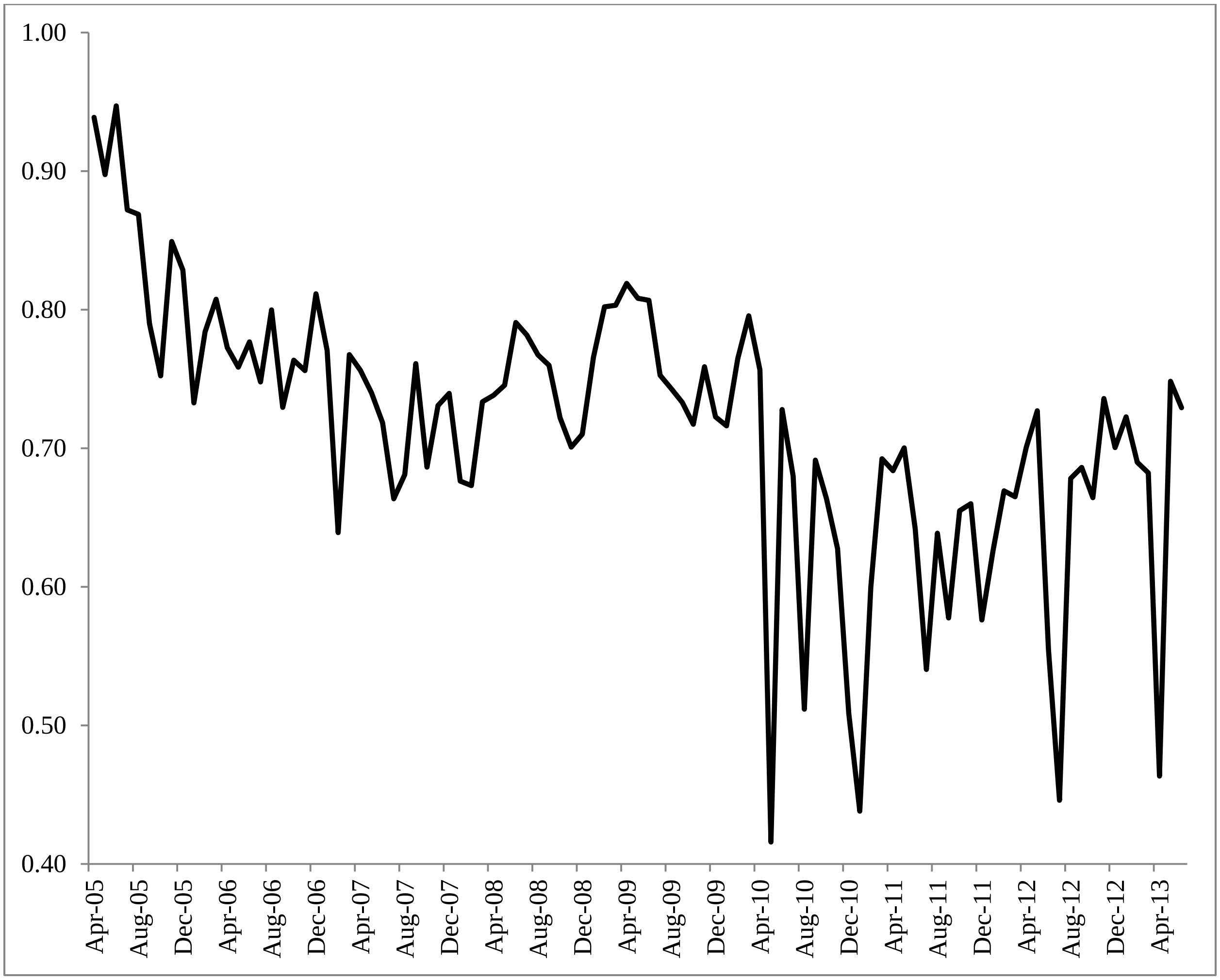


**Fig. 1.** Monthly R-squared values from full sample cross-sectional estimation of implied volatility.

**Table 1**
Summary Statistics for the data under study.

| stats | N | mean | median | sd | skewness | kurtosis |
|---|---|---|---|---|---|---|
| iv3m | 207,982 | 47.66 | 39.96 | 33.11 | 5.08 | 54.39 |
| hv90 | 291,275 | 39.83 | 34.55 | 33.68 | 2.27 | 15.90 |
| hv180 | 291,275 | 40.05 | 35.68 | 32.69 | 1.79 | 10.98 |
| hv30 | 291,275 | 38.99 | 32.52 | 35.66 | 3.37 | 35.03 |
| iv3ml~12 | 174,865 | 48.58 | 40.90 | 33.33 | 4.95 | 49.48 |
| lnmktcap | 246,197 | 7.36 | 7.29 | 1.88 | 0.03 | 3.21 |
| turn_bb | 271,426 | 1,320 | 233 | 10,400 | 0.00 | 0.00 |
| divyldw | 291,275 | 1.16 | 0.00 | 2.52 | 3.19 | 14.59 |
| beta | 291,275 | 1.14 | 1.07 | 6.99 | 223 | 61,091 |
| npm_obs | 249,919 | -2.84 | 0.06 | 121 | -29.89 | 17,670 |
| peg | 31,804 | 4.14 | 1.09 | 93.46 | 67.55 | 5,197 |
| turn | 268,864 | 0.29 | 0.15 | 28.69 | 427 | 191,390 |
| roa | 255,534 | 0.01 | 0.01 | 3.00 | 173 | 33,484 |
| roe | 254,965 | 0.03 | 0.03 | 9.35 | -109.27 | 27,763 |
| ltg | 190,837 | 0.02 | 0.00 | 6.28 | 0.00 | 0.10 |
| ltd_eq | 255,731 | 1.42 | 0.29 | 229 | 183 | 38,846 |
| totdbt~q | 254,802 | 3.09 | 1.03 | 310 | 167 | 33,808 |
| resid | 213,298 | 0.03 | 0.22 | 146 | 208 | 108,953 |
| xsret12 | 240,314 | 0.00 | 0.00 | 0.01 | 0.00 | 0.07 |
| p_fcf_bb | 156,067 | 168 | 16.72 | 18,776 | 192 | 37,541 |
| mktbook | 70,147 | 2.45 | 0.85 | 65.63 | 8.72 | 11,899 |
| vol_put | 223,304 | 29,238 | 2,028 | 141,248 | 20.12 | 691 |
| tang | 194,509 | 86.57 | 8.05 | 848 | 52.24 | 4,444 |
| volume | 237,151 | 0.42 | 0.12 | 1.84 | 0.00 | 0.00 |
| p_sale~b | 230,765 | 24.43 | 1.60 | 699 | 82.94 | 9,301 |
| vol_cal | 223,847 | 45,873 | 3,678 | 223,100 | 18.27 | 533 |
| ltg_sd | 788 | 9.18 | 5.00 | 16.08 | 6.85 | 59.50 |
| put_oi | 218,121 | 37,487 | 3,679 | 227,876 | 37.94 | 1,981 |
| xsret1 | 273,219 | 0.91 | 0.00 | 44.73 | 374 | 170,407 |
| os | 194,666 | 0.17 | 0.05 | 4.35 | 159 | 29,596 |
| tot_oi~K | 217,923 | 82.88 | 9.95 | 408 | 27.06 | 1,089 |
| pe_bb | 186,628 | 40.58 | 18.11 | 1,318 | 212 | 46,207 |
| tot_op~K | 218,116 | 3.77 | 0.17 | 42.83 | 57.37 | 4,313 |
| cal_oi | 219,127 | 45,111 | 5,698 | 197,576 | 23.07 | 961 |
| put_turn | 217,706 | 0.91 | 0.63 | 16.48 | 237 | 66,595 |
| cal_turn | 218,841 | 0.98 | 0.71 | 4.78 | 106 | 15,020 |
| eps_sd~t | 199,168 | 0.06 | 0.06 | 3.50 | 5.38 | 17,840 |
| opt_turn | 217,029 | 0.04 | 0.02 | 0.22 | 183 | 46,653 |

**Table 2**

Correlation matrix of explanatory variables, sorted by their absolute correlation with three-month implied volatility.

| | iv3m | hv90 | hv180 | hv30 | mktcap | $turn | divyld | beta | npm | peg | turn | roa | roe | ltg | ltd/e | totdbt | resid | xs12 | p_fcf | mktbook | vol_put | tang |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| hv90 | 0.88 | 1.00 | | | | | | | | | | | | | | | | | | | | |
| hv180 | 0.86 | 0.94 | 1.00 | | | | | | | | | | | | | | | | | | | |
| hv30 | 0.83 | 0.92 | 0.86 | 1.00 | | | | | | | | | | | | | | | | | | |
| mktcap | -0.61 | -0.50 | -0.50 | -0.46 | 1.00 | | | | | | | | | | | | | | | | | |
| $turn | -0.41 | -0.29 | -0.30 | -0.26 | 0.86 | 1.00 | | | | | | | | | | | | | | | | |
| divyld | -0.39 | -0.06 | -0.06 | -0.06 | 0.27 | 0.26 | 1.00 | | | | | | | | | | | | | | | |
| beta | 0.36 | 0.56 | 0.57 | 0.54 | -0.16 | -0.07 | 0.03 | 1.00 | | | | | | | | | | | | | | |
| npm | -0.34 | -0.25 | -0.26 | -0.23 | 0.37 | 0.30 | 0.20 | -0.11 | 1.00 | | | | | | | | | | | | | |
| peg | -0.33 | -0.32 | -0.32 | -0.29 | 0.06 | -0.01 | 0.26 | -0.21 | 0.03 | 1.00 | | | | | | | | | | | | |
| turn | 0.31 | 0.28 | 0.27 | 0.30 | 0.04 | 0.48 | 0.02 | 0.16 | -0.02 | -0.15 | 1.00 | | | | | | | | | | | |
| roa | -0.29 | -0.20 | -0.22 | -0.19 | 0.34 | 0.33 | 0.11 | -0.07 | 0.73 | -0.11 | 0.07 | 1.00 | | | | | | | | | | |
| roe | -0.29 | -0.22 | -0.23 | -0.20 | 0.35 | 0.31 | 0.12 | -0.10 | 0.58 | -0.10 | 0.02 | 0.73 | 1.00 | | | | | | | | | |
| ltg | 0.25 | 0.21 | 0.18 | 0.20 | -0.18 | -0.11 | -0.42 | 0.11 | -0.07 | -0.56 | 0.10 | 0.09 | 0.00 | 1.00 | | | | | | | | |
| ltd/e | -0.22 | -0.14 | -0.13 | -0.13 | 0.28 | 0.23 | 0.30 | 0.00 | 0.03 | 0.09 | 0.00 | -0.06 | 0.06 | -0.33 | 1.00 | | | | | | | |
| totdbt | -0.21 | -0.14 | -0.13 | -0.13 | 0.27 | 0.22 | 0.29 | -0.02 | 0.02 | 0.07 | -0.02 | -0.08 | 0.10 | -0.32 | 0.80 | 1.00 | | | | | | |
| resid | -0.20 | -0.15 | -0.14 | -0.14 | 0.34 | 0.27 | 0.23 | -0.01 | 0.12 | 0.10 | 0.02 | 0.13 | 0.11 | -0.26 | 0.35 | 0.22 | 1.00 | | | | | |
| xs12 | -0.14 | -0.11 | -0.11 | -0.10 | 0.18 | 0.16 | 0.00 | -0.04 | 0.17 | 0.11 | 0.03 | 0.18 | 0.18 | 0.06 | 0.02 | 0.02 | 0.02 | 1.00 | | | | |
| p_fcf | -0.12 | -0.12 | -0.13 | -0.10 | 0.08 | 0.05 | -0.14 | -0.04 | 0.06 | 0.19 | -0.03 | 0.14 | 0.04 | 0.29 | -0.16 | -0.24 | 0.03 | 0.12 | 1.00 | | | |
| mktbook | 0.11 | 0.26 | 0.27 | 0.22 | 0.13 | 0.40 | 0.06 | 0.22 | 0.09 | -0.04 | 0.57 | 0.22 | 0.13 | 0.25 | 0.09 | 0.10 | -0.06 | 0.13 | 0.20 | 1.00 | | |
| vol_put | -0.11 | -0.02 | -0.04 | 0.00 | 0.58 | 0.79 | 0.07 | 0.01 | 0.17 | -0.11 | 0.44 | 0.20 | 0.20 | 0.03 | 0.07 | 0.08 | 0.11 | 0.09 | 0.03 | 0.39 | 1.00 | |
| tang | 0.10 | 0.08 | 0.08 | 0.07 | -0.06 | -0.07 | 0.09 | 0.06 | 0.06 | 0.03 | -0.01 | -0.07 | 0.01 | -0.07 | -0.02 | 0.12 | 0.19 | 0.00 | 0.03 | -0.06 | 0.00 | 1.00 |
| volume | -0.10 | -0.02 | -0.02 | 0.01 | 0.65 | 0.86 | 0.14 | 0.06 | 0.12 | -0.05 | 0.56 | 0.12 | 0.14 | -0.12 | 0.18 | 0.17 | 0.22 | 0.02 | -0.06 | 0.34 | 0.73 | -0.03 |
| p_sales | -0.09 | -0.11 | -0.10 | -0.10 | 0.07 | 0.03 | -0.07 | -0.10 | 0.29 | 0.28 | -0.04 | 0.04 | 0.00 | 0.19 | -0.23 | -0.31 | -0.22 | 0.16 | 0.37 | 0.24 | 0.06 | 0.04 |
| vol_cal | -0.09 | -0.01 | -0.02 | 0.01 | 0.56 | 0.77 | 0.05 | 0.03 | 0.15 | -0.10 | 0.43 | 0.18 | 0.18 | 0.03 | 0.06 | 0.06 | 0.11 | 0.11 | 0.04 | 0.39 | 0.93 | 0.00 |
| ltg_sd | 0.09 | 0.11 | 0.13 | 0.11 | 0.14 | 0.12 | -0.09 | 0.23 | -0.07 | -0.39 | -0.09 | -0.11 | -0.05 | 0.15 | 0.01 | -0.08 | 0.26 | 0.10 | -0.08 | -0.24 | 0.16 | 0.08 |
| put_oi | -0.09 | -0.02 | -0.03 | -0.01 | 0.57 | 0.76 | 0.06 | 0.01 | 0.13 | -0.12 | 0.37 | 0.15 | 0.17 | 0.00 | 0.09 | 0.10 | 0.11 | 0.05 | 0.01 | 0.35 | 0.94 | -0.01 |
| xs1 | -0.09 | -0.03 | -0.03 | -0.04 | 0.07 | 0.04 | 0.00 | -0.02 | 0.02 | 0.07 | -0.04 | 0.03 | 0.02 | 0.00 | 0.01 | 0.01 | 0.01 | 0.15 | 0.06 | -0.01 | 0.00 | -0.01 |
| os | -0.06 | -0.02 | -0.02 | -0.01 | 0.35 | 0.48 | -0.03 | 0.00 | 0.14 | -0.08 | 0.27 | 0.21 | 0.19 | 0.12 | -0.05 | -0.05 | 0.03 | 0.13 | 0.11 | 0.35 | 0.82 | 0.03 |
| tot_oi | -0.05 | 0.01 | 0.00 | 0.01 | 0.55 | 0.75 | 0.04 | 0.03 | 0.11 | -0.12 | 0.38 | 0.12 | 0.15 | 0.00 | 0.07 | 0.09 | 0.11 | 0.05 | 0.01 | 0.35 | 0.93 | -0.01 |
| pe | -0.05 | -0.07 | -0.07 | -0.06 | -0.02 | -0.06 | -0.19 | -0.01 | -0.10 | 0.28 | -0.05 | -0.11 | -0.22 | 0.30 | -0.13 | -0.21 | -0.07 | 0.19 | 0.48 | 0.15 | -0.05 | -0.03 |
| opt_vol | -0.04 | 0.01 | 0.00 | 0.03 | 0.50 | 0.69 | 0.03 | 0.03 | 0.13 | -0.10 | 0.36 | 0.15 | 0.16 | 0.03 | 0.04 | 0.06 | 0.09 | 0.09 | 0.04 | 0.34 | 0.85 | 0.01 |
| cal_oi | -0.04 | 0.02 | 0.01 | 0.02 | 0.53 | 0.73 | 0.03 | 0.04 | 0.10 | -0.11 | 0.38 | 0.11 | 0.14 | 0.01 | 0.06 | 0.07 | 0.11 | 0.05 | 0.01 | 0.35 | 0.89 | -0.01 |
| put_turn | 0.04 | 0.09 | 0.05 | 0.15 | 0.17 | 0.32 | -0.02 | 0.02 | 0.10 | -0.05 | 0.35 | 0.17 | 0.13 | 0.11 | -0.05 | -0.04 | -0.01 | 0.12 | 0.08 | 0.24 | 0.47 | 0.01 |
| cal_turn | -0.03 | 0.02 | 0.01 | 0.08 | 0.21 | 0.35 | 0.00 | 0.02 | 0.13 | -0.02 | 0.32 | 0.19 | 0.16 | 0.09 | -0.02 | -0.02 | 0.01 | 0.20 | 0.10 | 0.24 | 0.41 | 0.02 |
| sdest | 0.02 | 0.03 | 0.03 | 0.03 | 0.02 | 0.01 | 0.05 | 0.04 | 0.10 | -0.05 | 0.03 | 0.07 | 0.03 | -0.04 | 0.10 | 0.06 | 0.18 | 0.02 | -0.05 | -0.07 | -0.01 | 0.09 |
| opt_turn | 0.02 | 0.01 | 0.00 | 0.04 | 0.08 | 0.13 | 0.00 | 0.00 | 0.07 | -0.01 | 0.09 | 0.10 | 0.08 | 0.06 | -0.04 | -0.03 | -0.01 | 0.09 | 0.06 | 0.09 | 0.14 | 0.03 |

**Table 2 (continued)**

Correlation matrix of explanatory variables, sorted by their absolute correlation with three-month implied volatility (continued).

| | volume | p_sales | vol_cal | ltg_sd | put_oi | xs1 | os | tot_oi | pe | opt_vol | cal_oi | put_turn | cal_turn | sdest |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| p_sales | -0.01 | 1.00 | | | | | | | | | | | | |
| vol_cal | 0.75 | 0.08 | 1.00 | | | | | | | | | | | |
| ltg_sd | 0.19 | -0.10 | 0.08 | 1.00 | | | | | | | | | | |
| put_oi | 0.74 | 0.04 | 0.90 | 0.15 | 1.00 | | | | | | | | | |
| xs1 | 0.00 | 0.05 | 0.05 | 0.02 | 0.01 | 1.00 | | | | | | | | |
| os | 0.33 | 0.14 | 0.83 | 0.09 | 0.75 | 0.04 | 1.00 | | | | | | | |
| tot_oi | 0.76 | 0.05 | 0.94 | 0.11 | 0.97 | 0.01 | 0.76 | 1.00 | | | | | | |
| pe | -0.11 | 0.44 | -0.04 | 0.07 | -0.06 | 0.11 | 0.02 | -0.06 | 1.00 | | | | | |
| opt_vol | 0.66 | 0.08 | 0.86 | 0.13 | 0.83 | 0.04 | 0.72 | 0.84 | -0.03 | 1.00 | | | | |
| cal_oi | 0.76 | 0.06 | 0.94 | 0.09 | 0.93 | 0.01 | 0.75 | 0.99 | -0.06 | 0.83 | 1.00 | | | |
| put_turn | 0.23 | 0.07 | 0.36 | 0.05 | 0.20 | -0.01 | 0.42 | 0.20 | 0.03 | 0.33 | 0.21 | 1.00 | | |
| cal_turn | 0.25 | 0.10 | 0.49 | 0.04 | 0.25 | 0.13 | 0.50 | 0.23 | 0.05 | 0.37 | 0.22 | 0.58 | 1.00 | |
| sdest | 0.01 | -0.12 | -0.01 | 0.19 | -0.02 | 0.00 | -0.01 | -0.02 | 0.01 | -0.01 | -0.02 | 0.02 | 0.02 | 1.00 |
| opt_turn | 0.07 | 0.06 | 0.14 | 0.06 | 0.04 | 0.07 | 0.17 | 0.03 | 0.03 | 0.53 | 0.03 | 0.32 | 0.35 | 0.02 |

**Table 3**
Implied volatility estimation models.

| | Full Model Estimation (ex post results) | | Estimation to May 2012 for Out of Sample Testing | |
|---|---|---|---|---|
| Variable | Coefficient (t-stat) | | Coefficient (t-stat) | |
| 90 day historical volatility | 0.25 | *** | 0.26 | *** |
| | (13.67) | | (12.87) | |
| 180 day historical volatility | 0.33 | *** | 0.31 | *** |
| | (15.34) | | (13.67) | |
| 30 day historical volatility | 0.16 | *** | 0.16 | *** |
| | (10.59) | | (9.67) | |
| Market Cap | -1.31 | *** | -1.25 | *** |
| | (-11.10) | | (-9.53) | |
| Dividend Yield | -0.51 | *** | -0.50 | *** |
| | (-11.63) | | (-10.12) | |
| Net Profit Margin | -0.73 | * | -0.83 | * |
| | (-1.90) | | (-1.90) | |
| Stock Turnover Ratio | 1.75 | *** | 2.18 | *** |
| | (2.77) | | (3.34) | |
| Dollar Stock Turnover | -1.01 | *** | -0.94 | *** |
| | (-5.00) | | (-4.22) | |
| Est. Long Term Growth Rate | 0.02 | *** | 0.02 | *** |
| | (3.25) | | (3.19) | |
| 1 Month Excess Returns | -0.08 | *** | -0.08 | *** |
| | (-9.26) | | (-8.34) | |
| Total Option Volume | 0.06 | *** | 0.06 | *** |
| | (12.45) | | (11.66) | |
| Put Turnover Ratio | -0.33 | *** | -0.40 | *** |
| | (-3.64) | | (-4.28) | |
| Total Option Turnover Ratio | 2.30 | ** | 2.18 | ** |
| | (2.35) | | (1.94) | |
| Constant | 42.55 | *** | 40.67 | *** |
| | (10.70) | | (9.36) | |
| n (firm-months) | 167,762 | | 136,656 | |
| Number of Months | 102 | | 89 | |
| Average R-squared | 0.724 | | 0.734 | |

(*), (**), (***) indicate statistically significant factors at 10%, 5% and 1% levels respectively.

**Table 4**
Portfolios sorted on predicted values of implied volatility.

Panel A
Three month implied volatility sorted by percentage residuals from the Fama MacBeth (1973) two-step procedure.

| Full Estimation Model | | | Estimation to May 2012 | | |
|---|---|---|---|---|---|
| Decile | Mean | Std Dev | Decile | Mean | Std Dev |
| 1 | 67.22 | 0.34 | 1 | 68.09 | 0.84 |
| 2 | 48.44 | 0.18 | 2 | 45.83 | 0.37 |
| 3 | 45.02 | 0.15 | 3 | 40.47 | 0.29 |
| 4 | 43.67 | 0.14 | 4 | 37.76 | 0.27 |
| 5 | 42.76 | 0.14 | 5 | 35.78 | 0.24 |
| 6 | 41.58 | 0.14 | 6 | 34.43 | 0.24 |
| 7 | 40.80 | 0.14 | 7 | 33.23 | 0.23 |
| 8 | 39.69 | 0.14 | 8 | 31.19 | 0.23 |
| 9 | 38.29 | 0.14 | 9 | 29.50 | 0.24 |
| 10 | 37.38 | 0.17 | 10 | 29.26 | 0.31 |

Panel B
Next month change in three month implied volatility sorted by percentage residuals from the Fama MacBeth (1973) two-step procedure.

| Full Estimation Model | | | Estimation to May 2012 | | |
|---|---|---|---|---|---|
| Decile | Mean | Std Dev | Decile | Mean | Std Dev |
| 1 | 12.25 | 0.30 | 1 | 14.53 | 0.96 |
| 2 | 3.76 | 0.12 | 2 | 2.07 | 0.24 |
| 3 | 2.14 | 0.11 | 3 | -0.21 | 0.21 |
| 4 | 1.14 | 0.11 | 4 | -0.75 | 0.21 |
| 5 | 0.62 | 0.10 | 5 | -1.04 | 0.20 |
| 6 | 0.11 | 0.10 | 6 | -1.44 | 0.19 |
| 7 | -0.69 | 0.10 | 7 | -2.22 | 0.20 |
| 8 | -1.29 | 0.10 | 8 | -2.79 | 0.21 |
| 9 | -2.23 | 0.10 | 9 | -3.79 | 0.22 |
| 10 | -2.68 | 1.63 | 10 | -6.66 | 0.35 |
| Low-High | 14.94 | | Low-High | 21.19 | |
| t-stat | 19.79 | | t-stat | 40.15 | |
| n | 167,762 | | n | 31,106 | |

Panel C
Next month returns sorted by Deciles.

| Full Estimation Model | | | Estimation to May 2012 | | |
|---|---|---|---|---|---|
| Decile | Mean | Std Dev | Decile | Mean | Std Dev |
| 1 | 0.87 | 0.14 | 1 | 0.13 | 0.29 |
| 2 | 0.26 | 0.10 | 2 | 0.17 | 0.23 |
| 3 | 0.36 | 0.09 | 3 | 0.44 | 0.21 |
| 4 | 0.23 | 0.09 | 4 | 0.24 | 0.19 |
| 5 | 0.31 | 0.09 | 5 | 0.44 | 0.18 |
| 6 | 0.23 | 0.08 | 6 | 0.39 | 0.17 |
| 7 | 0.30 | 0.09 | 7 | 0.17 | 0.16 |
| 8 | 0.33 | 0.08 | 8 | 0.42 | 0.16 |
| 9 | 0.47 | 0.08 | 9 | 0.34 | 0.14 |
| 10 | 0.34 | 0.09 | 10 | 0.18 | 0.17 |
| Low-High | 0.53 | | Low-High | -0.05 | |
| t-stat | 3.95 | | t-stat | -0.18 | |
| n | 167,762 | | n | 31,106 | |

**Table 5**
Estimation of the spread between three month implied volatility and ninety day realized stock volatility.

| | Full Model Estimation (ex post results) | | Estimation to May 2012 for Out of Sample Testing | |
|---|---|---|---|---|
| | Coefficient | | Coefficient | |
| Variable | (t-stat) | | (t-stat) | |
| 90 day historical volatility | 0.24 | *** | 0.24 | *** |
| | (13.00) | | (12.14) | |
| 180 day historical volatility | 0.33 | *** | 0.31 | *** |
| | (15.34) | | (13.67) | |
| 30 day historical volatility | -0.83 | *** | -0.83 | *** |
| | (-41.04) | | (-36.19) | |
| Market Cap | -1.31 | *** | -1.25 | *** |
| | (-11.10) | | (-9.53) | |
| Dividend Yield | -0.51 | *** | -0.50 | *** |
| | (-11.63) | | (-10.12) | |
| Net Profit Margin | -0.73 | * | -0.83 | * |
| | (-1.90) | | (-1.90) | |
| Stock Turnover Ratio | 1.75 | *** | 2.18 | *** |
| | (2.77) | | (3.34) | |
| Dollar Stock Turnover | -1.01 | *** | -0.94 | *** |
| | (-5.00) | | (-4.22) | |
| Est. Long Term Growth Rate | 0.02 | *** | 0.02 | *** |
| | (3.25) | | (3.19) | |
| 1 Month Excess Returns | -0.08 | *** | -0.08 | *** |
| | (-9.26) | | (-8.34) | |
| Total Option Volume | 0.06 | *** | 0.06 | *** |
| | (12.45) | | (11.66) | |
| Put Turnover Ratio | -0.33 | *** | -0.40 | *** |
| | (-3.64) | | (-4.28) | |
| Total Option Turnover Ratio | 2.30 | ** | 2.18 | ** |
| | (2.35) | | (1.94) | |
| Constant | 42.53 | *** | 40.66 | *** |
| | (10.68) | | (9.35) | |
| n (firm-months) | 167,762 | | 136,656 | |
| Number of Months | 102 | | 89 | |
| Average R-squared | 0.581 | | 0.593 | |

(*), (**), (***) indicate statistically significant factors at 10%, 5% and 1% levels respectively.

**Table 6**
Portfolios sorted on predicted values of the spread between three month implied volatility and ninety day realized stock volatility.

Panel A
Difference between current three month implied volatility and three-month historical volatility.

| Full Estimation Model | | | Estimation to May 2012 | | |
|---|---|---|---|---|---|
| Decile | Mean | Std Dev | Decile | Mean | Std Dev |
| 1 | -21.21 | 0.24 | 1 | -18.76 | 0.55 |
| 2 | -5.67 | 0.15 | 2 | -0.90 | 0.30 |
| 3 | -1.64 | 0.14 | 3 | 3.10 | 0.34 |
| 4 | 0.89 | 0.13 | 4 | 6.14 | 0.45 |
| 5 | 2.88 | 0.12 | 5 | 7.32 | 0.31 |
| 6 | 4.44 | 0.10 | 6 | 8.24 | 0.22 |
| 7 | 5.70 | 0.09 | 7 | 8.59 | 0.20 |
| 8 | 7.33 | 0.10 | 8 | 9.68 | 0.21 |
| 9 | 8.79 | 0.09 | 9 | 9.89 | 0.18 |
| 10 | 10.58 | 0.09 | 10 | 10.32 | 0.19 |

Panel B
Next month difference between current three month implied volatility and three-month historical volatility.

| Full Estimation Model | | | Estimation to May 2012 | | |
|---|---|---|---|---|---|
| Decile | Mean | Std Dev | Decile | Mean | Std Dev |
| 1 | -5.37 | 0.22 | 1 | -1.28 | 0.53 |
| 2 | -1.13 | 0.17 | 2 | 3.48 | 0.39 |
| 3 | 0.18 | 0.16 | 3 | 4.96 | 0.44 |
| 4 | 0.77 | 0.14 | 4 | 5.53 | 0.32 |
| 5 | 1.75 | 0.15 | 5 | 5.94 | 0.48 |
| 6 | 2.48 | 0.14 | 6 | 5.77 | 0.27 |
| 7 | 2.71 | 0.13 | 7 | 5.51 | 0.27 |
| 8 | 3.12 | 0.13 | 8 | 5.78 | 0.38 |
| 9 | 3.75 | 0.12 | 9 | 5.94 | 0.33 |
| 10 | 4.58 | 0.15 | 10 | 5.92 | 0.31 |

Panel C
Change in spread between three-month implied volatility and 90 day realized volatility.

| Full Estimation Model | | Estimation to May 2012 | |
|---|---|---|---|
| Decile | Change | Decile | Change |
| 1 | 15.84 | 1 | 17.48 |
| 2 | 4.54 | 2 | 4.37 |
| 3 | 1.82 | 3 | 1.86 |
| 4 | -0.12 | 4 | -0.60 |
| 5 | -1.13 | 5 | -1.38 |
| 6 | -1.97 | 6 | -2.47 |
| 7 | -2.99 | 7 | -3.08 |
| 8 | -4.20 | 8 | -3.91 |
| 9 | -5.03 | 9 | -3.96 |
| 10 | -6.00 | 10 | -4.40 |
| Low-High | 21.83 | Low-High | 21.88 |

Panel D
Next month returns sorted by deciles based on the change in spread between three-month implied volatility and 90 day realized volatility.

| Full Estimation Model | | | Estimation to May 2012 | | |
|---|---|---|---|---|---|
| Decile | Mean | Std Dev | Decile | Mean | Std Dev |
| 1 | 0.45 | 0.12 | 1 | 0.10 | 0.23 |
| 2 | 0.38 | 0.10 | 2 | 0.44 | 0.22 |
| 3 | 0.17 | 0.10 | 3 | 0.13 | 0.19 |
| 4 | 0.40 | 0.09 | 4 | 0.14 | 0.20 |
| 5 | 0.21 | 0.09 | 5 | 0.27 | 0.19 |
| 6 | 0.39 | 0.09 | 6 | 0.73 | 0.19 |
| 7 | 0.37 | 0.09 | 7 | 0.14 | 0.18 |
| 8 | 0.54 | 0.09 | 8 | 0.21 | 0.18 |
| 9 | 0.41 | 0.08 | 9 | 0.54 | 0.18 |
| 10 | 0.38 | 0.09 | 10 | 0.20 | 0.19 |
| Low-High | 0.07 | | Low-High | -0.10 | |
| t-stat | 0.50 | | t-stat | -0.78 | |
| n | 167,762 | | n | 31,106 | |

**Table 7**
Portfolios sorted on the actual values of the spread between three month implied volatility and ninety day realized stock volatility.

Panel A
Difference between current three month implied volatility and three-month historical volatility.

| Full Estimation Model | | | Estimation to May 2012 | | |
|---|---|---|---|---|---|
| Decile | Mean | Std Dev | Decile | Mean | Std Dev |
| 1 | -26.78 | 0.23 | 1 | -25.12 | 0.64 |
| 2 | -8.30 | 0.08 | 2 | -4.51 | 0.06 |
| 3 | -4.03 | 0.07 | 3 | -0.78 | 0.05 |
| 4 | -1.27 | 0.06 | 4 | 1.48 | 0.04 |
| 5 | 0.97 | 0.05 | 5 | 3.37 | 0.04 |
| 6 | 3.06 | 0.04 | 6 | 5.29 | 0.04 |
| 7 | 5.32 | 0.04 | 7 | 7.59 | 0.05 |
| 8 | 8.24 | 0.04 | 8 | 10.91 | 0.05 |
| 9 | 13.25 | 0.04 | 9 | 17.16 | 0.07 |
| 10 | 46.55 | 0.39 | 10 | 50.58 | 0.83 |

Panel B
Next month difference between current three month implied volatility and three-month historical volatility.

| Full Estimation Model | | | Estimation to May 2012 | | |
|---|---|---|---|---|---|
| Decile | Mean | Std Dev | Decile | Mean | Std Dev |
| 1 | -7.15 | 0.22 | 1 | -3.42 | 0.60 |
| 2 | -2.26 | 0.12 | 2 | 1.44 | 0.23 |
| 3 | -1.04 | 0.10 | 3 | 2.04 | 0.16 |
| 4 | 0.04 | 0.09 | 4 | 2.75 | 0.16 |
| 5 | 0.66 | 0.08 | 5 | 3.15 | 0.17 |
| 6 | 1.47 | 0.08 | 6 | 3.63 | 0.17 |
| 7 | 2.36 | 0.09 | 7 | 5.11 | 0.21 |
| 8 | 3.69 | 0.11 | 8 | 6.89 | 0.39 |
| 9 | 6.17 | 0.14 | 9 | 10.47 | 0.42 |
| 10 | 32.89 | 0.44 | 10 | 35.91 | 1.00 |

Panel C
Change in spread between three-month implied volatility and 90 day realized volatility.

| Full Estimation Model | | Estimation to May 2012 | |
|---|---|---|---|
| Decile | Change | Decile | Change |
| 1 | 19.63 | 1 | 21.70 |
| 2 | 6.04 | 2 | 5.95 |
| 3 | 2.99 | 3 | 2.82 |
| 4 | 1.31 | 4 | 1.26 |
| 5 | -0.31 | 5 | -0.22 |
| 6 | -1.60 | 6 | -1.66 |
| 7 | -2.96 | 7 | -2.48 |
| 8 | -4.55 | 8 | -4.02 |
| 9 | -7.08 | 9 | -6.69 |
| 10 | -13.66 | 10 | -14.67 |
| Low-High | 33.29 | Low-High | 36.37 |

**Table 7 (continued)**
**Portfolios sorted on the actual values of the spread between three month implied volatility and ninety day realized stock volatility.**

Panel D
Next month change in three month implied volatility sorted by Deciles.

| Full Estimation Model | | | Estimation to May 2012 | | |
|---|---|---|---|---|---|
| Decile | Mean | Std Dev | Decile | Mean | Std Dev |
| 1 | 2.00 | 1.34 | 1 | 1.36 | 0.40 |
| 2 | 0.01 | 0.10 | 2 | 1.63 | 0.47 |
| 3 | -0.05 | 0.10 | 3 | 1.23 | 0.44 |
| 4 | -0.12 | 0.10 | 4 | 1.53 | 0.51 |
| 5 | 0.01 | 0.10 | 5 | 1.18 | 0.53 |
| 6 | 0.04 | 0.09 | 6 | -0.28 | 0.31 |
| 7 | 0.26 | 0.10 | 7 | -1.00 | 0.27 |
| 8 | 0.53 | 0.10 | 8 | -1.65 | 0.22 |
| 9 | 1.57 | 0.11 | 9 | -2.18 | 0.23 |
| 10 | 9.37 | 0.28 | 10 | -4.26 | 0.25 |
| Low-High | -7.37 | | Low-High | 5.62 | |
| t-stat | -11.80 | | t-stat | 5.23 | |
| n | 167,762 | | n | 31,106 | |

**Table 8**
Initial Principal Component Analysis using thirteen significant factors (n = 167,762). The cumulative percent of variance explained by the factors is denoted in the column titled “Cumulative.”

| Component | Eigenvalue | Difference | Proportion | Cumulative |
|---|---|---|---|---|
| Component 1 | 3.285 | 1.524 | 0.253 | 0.253 |
| Component 2 | 1.761 | 0.681 | 0.135 | 0.388 |
| Component 3 | 1.080 | 0.051 | 0.083 | 0.471 |
| Component 4 | 1.029 | 0.029 | 0.079 | 0.550 |
| Component 5 | 1.000 | 0.000 | 0.077 | 0.627 |
| Component 6 | 1.000 | 0.002 | 0.077 | 0.704 |
| Component 7 | 0.998 | 0.019 | 0.077 | 0.781 |
| Component 8 | 0.979 | 0.155 | 0.075 | 0.856 |
| Component 9 | 0.824 | 0.160 | 0.063 | 0.920 |
| Component 10 | 0.664 | 0.422 | 0.051 | 0.971 |
| Component 11 | 0.242 | 0.170 | 0.019 | 0.989 |
| Component 12 | 0.072 | 0.007 | 0.006 | 0.995 |
| Component 13 | 0.066 | . | 0.005 | 1.000 |

**Table 9**
Initial Principal Component Analysis using thirteen significant factors. Rotated (orthogonalized) factor pattern of initial component loadings (n = 167,762, factors greater than 0.40 are presented in bold).

| Variable | Comp1 | Comp2 | Comp3 | Comp4 | Comp5 | Comp6 | Comp7 | Comp8 | Unexplained |
|---|---|---|---|---|---|---|---|---|---|
| hv90 | **0.533** | -0.063 | -0.012 | 0.069 | -0.004 | 0.007 | 0.002 | 0.000 | 0.091 |
| hv180 | **0.504** | -0.088 | -0.006 | 0.104 | -0.008 | 0.014 | -0.005 | 0.000 | 0.156 |
| hv30 | **0.521** | -0.015 | -0.017 | 0.005 | 0.004 | -0.001 | 0.007 | -0.001 | 0.173 |
| lnmktcap | -0.146 | **0.580** | -0.047 | 0.095 | 0.006 | 0.011 | 0.003 | -0.002 | 0.152 |
| divyld | 0.001 | 0.002 | 0.000 | -0.004 | 0.000 | **1.000** | 0.000 | 0.000 | 0.001 |
| npm_obs | 0.001 | 0.001 | 0.002 | 0.000 | -0.001 | 0.000 | **1.000** | 0.000 | 0.000 |
| $turn | 0.400 | 0.301 | -0.017 | -0.233 | 0.037 | -0.025 | -0.003 | -0.001 | **0.421** |
| ln_turn_bb | 0.033 | **0.687** | -0.059 | 0.010 | 0.023 | 0.002 | 0.002 | -0.001 | 0.074 |
| ltg | 0.000 | -0.001 | -0.001 | 0.000 | 0.001 | 0.000 | 0.000 | **1.000** | 0.000 |
| xsret1 | 0.016 | 0.021 | 0.001 | **0.960** | 0.005 | -0.005 | 0.000 | 0.000 | 0.051 |
| tot_opt_v~1K | 0.086 | 0.285 | **0.507** | 0.003 | -0.121 | -0.001 | -0.010 | 0.009 | **0.526** |
| put_turn | 0.001 | 0.008 | 0.019 | 0.004 | **0.990** | 0.001 | -0.001 | 0.001 | 0.016 |
| opt_turn | -0.027 | -0.086 | **0.858** | 0.000 | 0.052 | 0.001 | 0.005 | -0.004 | 0.208 |

**Table 10**
Revised Principal Component Analysis using four components constructed from seven significant factors. Rotated (orthogonalized) factor pattern of initial component loadings (n = 167,762, factors greater than 0.40 are presented in bold).

| Variable | Component 1 | Component 2 | Component 3 | Component 4 | Unexplained |
|---|---|---|---|---|---|
| | "Realized Volatility" | "Size" | "Returns" | "Hedging Activity" | |
| hv90 | **0.562** | -0.061 | 0.011 | -0.004 | 0.065 |
| hv180 | **0.538** | -0.096 | 0.039 | -0.015 | 0.134 |
| hv30 | **0.534** | 0.023 | -0.023 | 0.010 | 0.158 |
| $turn | 0.297 | **0.624** | -0.020 | -0.010 | 0.252 |
| turn_bb | -0.144 | **0.758** | 0.183 | -0.111 | 0.173 |
| xsret1 | 0.019 | -0.149 | **0.905** | -0.366 | 0.011 |
| put_turn | -0.001 | 0.036 | 0.381 | **0.924** | 0.000 |

**Table 11**
Implied volatility estimation model using estimated PCA components as independent variables.

| | Full Model Estimation (ex post results) | |
|---|---|---|
| Principal Component | Coefficient (t-stat) | |
| "Realized Volatility" | 12.02 | *** |
| | (65.69) | |
| "Size" | -2.85 | *** |
| | (-10.35) | |
| "Returns" | -4.75 | *** |
| | (-4.49) | |
| "Hedging Activity" | -7.88 | *** |
| | (-2.80) | |
| n (firm-months) | 167,762 | |
| Number of Months | 102 | |
| Average R-squared | 0.692 | |

(*), (**), (***) indicate statistically significant factors at 10%, 5% and 1% levels respectively.

**Table 12**
Portfolios sorted on predicted values of implied volatility estimated using PCA factors.

Panel A
Next month change in three month implied volatility sorted by Deciles.

| Full Estimation Model | | |
|---|---|---|
| Decile | Mean | Std Dev |
| 1 | 13.63 | 0.37 |
| 2 | 3.84 | 0.13 |
| 3 | 2.25 | 0.11 |
| 4 | 1.30 | 0.10 |
| 5 | 0.58 | 0.10 |
| 6 | 0.02 | 0.10 |
| 7 | -0.59 | 0.10 |
| 8 | -1.03 | 0.10 |
| 9 | -1.76 | 0.11 |
| 10 | -2.08 | 1.62 |
| Low-High | 15.71 | |
| t-stat | 24.55 | |
| n | 167,762 | |

Panel B
Next month returns sorted by Deciles.

| Full Estimation Model | | |
|---|---|---|
| Decile | Mean | Std Dev |
| 1 | 0.93 | 0.15 |
| 2 | 0.25 | 0.10 |
| 3 | 0.33 | 0.09 |
| 4 | 0.37 | 0.09 |
| 5 | 0.35 | 0.08 |
| 6 | 0.32 | 0.08 |
| 7 | 0.33 | 0.08 |
| 8 | 0.29 | 0.07 |
| 9 | 0.40 | 0.07 |
| 10 | 0.40 | 0.08 |
| Low-High | 0.53 | |
| t-stat | 4.04 | |
| n | 167,762 | |