Parameter Assumptions in Normative Analyses of Household Financial Decisions

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Normative analyses of household financial decisions typically assume parameters of the household utility function. Some general issues on parameter assumptions for normative analysis are discussed in this study. We review selected normative household analyses appearing in finance and economics journals, examine the parameters assumed for risk aversion, intertemporal elasticity, and personal discount rates, and discuss justifications authors presented for the parameter values assumed. Moreover, articles cited in the normative articles for justification of utility function parameter values, and some other articles that have provided estimates of household parameter values are examined. None of the normative articles cited an independent estimate of the personal discount rate, instead, arbitrary assumptions were made for the discount rate. Most of the sources cited for relative risk aversion and for intertemporal elasticity had estimates based on aggregate patterns. We conclude with recommendations for parameter assumptions in normative analyses of household financial decisions.

INTRODUCTION

In order to prescribe actions or policies for individuals (e.g., Findley and Caliendo, 2008) or evaluate whether households are making mistakes (e.g., Calvet, Campbell, and Sondini, 2007; Cocco, Gomes, and Mahenhout, 2005) economists typically use expected utility analysis. A crucial part of normative household finance is choosing plausible utility functions and plausible parameters of utility functions. The goal is reasonable normative analysis (Campbell, 2006). An important question, as Poterba et al. (2003) and Brown and Poterba (2000) noted, is estimation of utility function parameters. The key preference parameter for static evaluations of risky choices based on the expected utility of wealth levels is the household's relative risk aversion level. To evaluate intertemporal savings/consumption decisions and derive optimal choices for how much to save or borrow, two additional important preference parameters are the elasticity of the intertemporal utility function and the personal discount rate, the rate at which the utility of future consumption is discounted.

Saving and Consumption: An Overview of Intertemporal Utility Optimization

There are analyses that investigate risky choices over time, considering spending consequences of investment and saving choices in a life cycle investment context, but it is simpler to first consider intertemporal spending choices with certain income and

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investment return projections. With the original life cycle model of savings, the objective is specified as maximizing lifetime utility of spending in each remaining period, and the typical assumption is that the utility of spending in each period is additive and independent. The assumption of independence is not as restrictive as one might think, because with typical utility functions, such as the natural log, allowing spending to drop to zero in one period has a utility of minus infinity. The original life cycle model and many analyses over the past 50 years have assumed maximization of lifetime utility. Freyland (2004) provides an example of the optimization framework, starting with the lifetime budget constraint:

$$\sum_{t=1}^{T} C_t (1+r)^{-t} \le A_0 + \sum_{t=1}^{T} Y_t (1+r)^{-t} \tag{1}$$

Where C_t is consumption in period t, the real interest rate is r, and the income in period t is Y_t. The maximization problem is:

$$\max_{\{C_{1...}C_T\}} \sum_{t=1}^{T} (1+\rho)^{-t} U(C_t) \tag{2}$$

Where the personal discount factor, ρ, is how much the individual discounts the utility of consumption one period in the future, and U is a utility function, typically assumed to have decreasing marginal utility of consumption.

With no uncertainty in future income, the optimal condition for spending growth rate can be expressed (Freyland, 2004) as:

$$\frac{U'(C_{t+1})}{U'(C_t)} = \left(\frac{1+\rho}{1+r}\right) \tag{3}$$

It is common to assume that the utility function has constant elasticity, which is similar mathematically to the assumption of a constant relative risk aversion utility function for risky investment choices. Some authors refer to the elasticity of marginal utility with respect to consumption, which we will denote as elasticity, ε, and other authors refer to the intertemporal substitution elasticity (e.g., Barsky et al., 1997), which we will denote as θ. The relationship between θ and ε is:

$$\varepsilon = -1 / \theta \tag{4}$$

If the intertemporal utility function has constant elasticity, ε, Hanna, Fan and Chang (1995) show that the optimal growth rate in spending can be approximated by:

$$G_c \approx \frac{r-\rho}{-\varepsilon} \tag{5}$$

If the real interest rate is equal to the personal discount rate, then we obtain the simple result of perfect consumption smoothing, in other words, the optimal consumption path over the future should be to have equal consumption in each future period. Hanna et al. (1995) show that a very simple life cycle model, with the assumption of a specific date of death, can produce the familiar increasing then decreasing pattern of assets to provide for constant consumption, and that the optimal saving pattern depends on the projected income pattern. In this type of simplified model, the utility function parameter of intertemporal elasticity does not affect optimal saving patterns. However, if we allow for the interest rate minus the personal discount rate to be non-zero, then the optimal consumption pattern, while smooth compared to income, has some changes, and the solution to the optimization problem is more complicated.

The Personal Discount Rate, ρ

The personal discount rate, ρ, is sometimes assumed in economic theory to be equal to the real rate of return (e.g., Findley and Caliendo, 2008), but that assumption originates from models of the economy based on a representative household. Other studies have attempted to infer personal discount rates from consumer choices, for instance, for models of household appliances with different levels of energy efficiency. However, for normative purposes, these studies may not be very appropriate, given the likelihood of consumer ignorance, lack of computational ability, and various household budget constraints that are difficult to assess.

An introspective approach can help illustrate the issue of estimating plausible values of ρ for normative applications. Imagine a world with a zero real interest rate. What value would you place on the utility of spending next year compared to the utility of spending this year? If your household and health situation will be the same, and you can imagine the utility from spending next year, it seems plausible that you should value the utility of spending next year the same as this year. On the other hand, if you are comparing the utility for consumption this year to the utility of spending 50 years from now, you might place a lower value on the future spending because of the probability that you would be dead then. For normative purposes Hanna et al. (1995) assumed that ρ should be based on the annual risk of death, so it would have a value of roughly 0.001 for somebody age 20, and the cumulative effect would be substantial in terms of decades of compounding the effects. They also assumed that ρ is related to planned changes in household size, so that a couple planning to have two children would value the utility of spending this year less than they value the utility of spending when both children have been born (c.f., White, 1978).

If we want to understand why households make particular financial decisions, understanding their actual personal discount rates is important. However, for normative purposes, the appropriate choice of a personal discount rate involves other considerations. Baumol (1968) discussed the social discount rate for evaluating public investments, and noted that the choice was related to how we value the welfare of future generations compared to our generation. Moore, Boardman, Vining, Weimer, and Greenberg (2004) reviewed values assumed for the social discount rate by public agencies. Discussion of social discount rates is often closely related to discussion of rates of return available on public investments, and similar issues should be considered in discussion of personal discount rates. If we asked two individuals about how much money today would be equivalent to $1,000 one year from now, the answers would be influenced by the interest rate or rate of return opportunities faced by each individual. The real interest rate a household should consider in its saving decisions depends on its assets. For a retired household in 2012 with only stable liquid financial assets, the inflation-adjusted aftertax rate of return is negative. If a household is saving for retirement in a tax-advantaged retirement form such as an Individual Retirement Account in the United States, then the expected inflation-adjusted return on the portfolio chosen could be considered the relevant interest rate if we ignore uncertainty, so, e.g., arithmetic real annual returns for the 1926 to 2007 period of 0.7% for T-bills, 2.4% for intermediate government bonds, and 9.0% for large company stocks (Morningstar, 2008, p. 120). On the other hand, if a household is carrying credit card debt and also has no investments for future goals, we might consider the real aftertax interest rate on the debt as its relevant interest rate, so the interest rate could be 18% or higher. Obviously in terms of optimal consumption growth and therefore the household's saving and credit decisions, the interest rate could make a substantial difference.

Risky Choices: An Overview of Utility Optimization for Uncertain Choices

Consider a one-time choice between two alternatives, one with certain wealth W_0, and the other with a probability p of a wealth level W_1 and a probability 1-p of a wealth level W_2. The expected utility of the first choice is:

$$EU = U(W_0) \tag{6}$$

The expected utility of the second choice is:

$$EU = pU(W_1) + (1-p)\,U(W_2) \tag{7}$$

There is a class of constant relative risk aversion utility functions (CRRA) commonly used in expected utility analyses. The natural log utility function has a relative risk aversion level of 1, and can be expressed as:

$$U(W) = Ln(W) \tag{8}$$

For relative risk aversion levels other than 1, one class of CRRA utility functions can be expressed as:

$$U(W) = W^{(1-\lambda)} / (1- \lambda) \qquad (9)$$

Where λ is the coefficient of relative risk aversion.

Mathematically, for a constant relative risk aversion utility function, the coefficient of relative risk aversion, λ, is equal to $-\varepsilon$, where ε is the elasticity of marginal utility with respect to consumption for intertemporal choices.

As an example of using expected utility with this type of utility function to evaluate choices, assume a safe alternative with a wealth level of $1,000,000, and a risky alternative with a 70% chance of a wealth level of $1,200,000 and a 30% chance of a wealth level of $800,000. Which alternative has higher expected utility? For a relative risk aversion level of 1.0 (natural log utility function), the risky alternative has higher expected utility, and even for relative risk aversion levels of 2.0, 3.0, and 4.0, expected utility is higher for the risky alternative. However, for relative risk aversion levels of 5.0 or higher, the safe alternative has higher expected utility. How the relative risk aversion levels of individuals is measured? Kimball (1988) presented hypothetical income gambles to infer relative risk aversion, and we will discuss other approaches later in this paper.

Relative Risk Aversion and Intertemporal Elasticity

Expected utility of risky choices with standard constant relative risk aversion utility functions is mathematically similar to expected utility over future periods, and the relative risk aversion parameter is plausibly the same as the negative of the elasticity of marginal utility with respect to consumption, or, $-\varepsilon$. As we shall see in reviewing normative analyses of household financial decisions, many authors have assumed that both for intertemporal choices and risky choices, the same value of the parameter can be used. Other authors have used the assumptions of Epstein and Zin (1989), and allow for the parameters to be different. For an intertemporal choice, if one values the utility of consumption in period 1 as highly as the utility of consumption in period 2, conceptually this may seem to be the same as the expected utility for a state of the world with probability p versus a state of the world with probability 1-p. In both models, ultimately the decision will affect potential consumption, so the premise of standard utility functions, that there is consistent decreasing marginal utility of consumption, would seem to apply in both the intertemporal choice and the risky choice decisions. However, with personal discounting of future utility, the equivalence does not seem as clear, so the more complex approach suggested by Epstein and Zinn might seem reasonable.

Our Methods

To select normative personal finance articles, we used a sequential search based on Campbell's (2006) citations of normative articles, and selected some normative articles in economics and finance journals, and in some cases found articles that cited the earlier articles. Table I shows the selected normative articles, sorted in chronological order. The normative articles dealt with derivation of optimal portfolio choices (OP), optimal saving decisions (OS), optimal annuity decisions, and personal bankruptcy choices. For each of the 16 normative articles, we attempted to identify the author's assumptions about three parameters: intertemporal elasticity, relative risk aversion, and the personal discount rate. Intertemporal elasticity is listed as -ε to be equivalent to relative risk aversion. If the author cited a source for justification of any of these parameters, we analyzed that source to ascertain whether the source publication contained an independent estimation of the parameter or simply asserted the value. If a normative article did not cite the source for a particular parameter, we noted "No specific reference" in Table I. We did not attempt to assess assumptions in the normative articles other than the three parameter values.

SELECTED NORMATIVE STUDIES

Appendix 1 has more detailed discussions of each of the normative articles in Table I, but below is an overview of the articles. As shown in Table I, most of the normative articles derived optimal portfolio allocations and optimal savings levels at the same time in their specified model. In order to evaluate the sensitivity of portfolio allocations and consumption/saving rules, many of the articles used several different levels of relative risk aversion coefficients and elasticity of intertemporal substitution. For example, Campbell and Viceira (1999) used eight levels of relative risk aversion (0.75 to 40). The discount rate was set to 4% to 6% in most of the articles.

Some normative articles derived optimal savings levels and examined predictions of a life-cycle simulation model, such as Hubbard, Skinner, and Zeldes (1994); Hubbard, Skinner, and Zeldes (1995); Brown and Poterba (2000); and Scholz, Seshadri, and Khitatrakun (2006). In those articles, the preference parameters in the base case were typically set as: the coefficient of relative risk aversion was 3, the elasticity of intertemporal substitution was 3, and the discount rate was 4% per year.

Optimal portfolio allocations were derived by some normative articles such as Cocco (2005); Gomes and Michaelides (2005); Gomes and Michaelides (2008); and Horneff, Maurer, and Stamos (2008d). In those articles, the preference parameters in the base case were typically set as: the coefficient of relative risk aversion was 5, the elasticity of intertemporal substitution was 5, and the discount rate was 4% per year. In addition, two normative articles regarding personal bankruptcy issue were selected: Livshits, Macgee and Tertil (2007) and Lopes (2008). In Livshits et al., relative risk aversion was assumed to be 2 and the personal discount rate was assumed to be 6% per year, while in Lopes, relative risk aversion was assumed to be 3 and the personal discount rate was assumed to be 3% per year.

Estimates of Parameters

Thirteen of the normative articles listed in Table I assumed one or more values of the intertemporal elasticity, and 12 of the articles cited a source for justification of the value, although some of the sources were other normative studies that assumed a value without specific evidence for that value. Cocco et al. (2005) cited Mehra and Prescott (1985) for the value assumed for intertemporal elasticity, even though that study's estimation based on the equity premium was really for relative risk aversion. Campbell and Viceira (1999) and Campbell et al. (2001) cited Hall (1988), which had estimates of intertemporal elasticity based on aggregate consumption patterns. Hubbard et al. (1994; 1995) cited Skinner (1985), which had estimates based on changes in consumption over time by individual households. None of the normative studies cited independent estimates of the personal discount rate, other than citations related to discounting based on mortality tables. The citations for the personal discount rate independent of discounting for the risk of death were all of studies that had made arbitrary assertions of plausible rates.

ESTIMATES OF INTERTEMPORAL ELASTICITY

Table II contains estimates of intertemporal elasticity, including studies cited by the normative articles listed in Table I and also some other studies. Many of the normative studies in Table I cited the same study for intertemporal elasticity and for relative risk aversion, so Appendix 2 has a more detailed discussion of the articles in Table II and in Table III. Equation 5 in this paper shows that optimal consumption growth depends on the real interest rate, the personal discount factor, and the intertemporal elasticity. If we can observe actual consumption growth, or if responses choose between sets of hypothetical consumption paths, we can infer intertemporal elasticity if we know or assume a value for the real interest rate and if we assume some value for the personal discount factor. For instance, estimates of intertemporal elasticity could be based on aggregate consumption growth, given some assumption about the real interest rate and the discount rate. Estimates of intertemporal elasticity ranged from 3 to 6 (Table II).

ESTIMATES OF RELATIVE RISK AVERSION

Table III contains estimates of relative risk aversion, including studies cited by the normative articles listed in Table I and also some other studies. Appendix 2 has a more detailed discussion of the articles in Table III. Some estimates were based on actual investment choices, which could be problematic. As Hanna, Waller, and Finke (2008) note, investment choices may be based on risk tolerance, but also risk capacity and expectations. Two individuals with identical risk aversion but different expectations might choose different portfolios. Also, at some points of the household life cycle, it may be optimal to have no investments, so the fact that a household owns no risky assets does not necessarily indicate high risk aversion. Carefully worded hypothetical questions may control for other factors that determine actual investment choices, and therefore may reveal each respondent's true risk aversion. The estimates based on hypothetical scenarios range from 4.4 to 6, although considerable individual variation is present, for instance, in a student sample (Hanna and Lindamood, 2008), an interquartile range from 2 to 6, and in a sample of older adults (Fang and Hanna, 2008), substantial proportions having levels of relative risk aversion under 3.8 or over 7.5. Some estimates based on aggregate market behavior, for instance, the equity premium (Mehra and Prescott, 1985), are similar to the estimates based on individual responses to hypothetical questions.

ESTIMATES OF THE PERSONAL DISCOUNT RATE

The normative studies in Table I did not cite any independent sources of the personal discount rate, so other studies that estimated the personal discount rate based on actual behavior and hypothetical choices were reviewed (Table IV). We followed Federick, Lowenstein and O'Donoghue (2002)'s classification of two types of estimates: (1) field studies and (2) experimental studies. Appendix 3 includes more detailed summaries of each study in Table IV. Frederick et al. (2002) noted that in field studies, researchers estimate the discount rate from actual economic decisions by individuals. Moreoever, some studies have also attempted to estimate the discount rate from aggregate macroeconomic patterns. Four articles using the field study approach were selected: Lawrance (1991); Carroll and Samwick (1997); Samwick (1998); and Gourinchas and Parker (2003). Frederick et al. (2002) also mentioned a second way to estimate the discount rate, by experimental studies in which people evaluate real or hypothetical outcomes. We reviewed three experimental studies: Barsky, et al. (1997); Coller and Williams (1999); and Harrison and Williams (2002).

As shown in Table IV, there was a huge variation of estimates of the personal discount rate across studies. For example, Barsky, et al. (1997) estimated the discount rate to be 0.8% per year while Harrison and Williams (2002) reported a discount rate of 28% per year. In all such estimates, modeling the decision, including incorporating the interest rate or rate of return faced by individuals, is crucial in obtaining a valid estimate. For instance, if individuals were asked whether they would prefer $1,000 a year in the future or $900 today, it would be important to know whether they had credit card balances or if the relevant interest rate was the aftertax inflation-adjusted interest rate on passbook savings. Two individuals with identical time preferences might give different responses to the hypothetical question depending on their financial constraints.

Even if some individuals behave as if they have high personal discount rates, for normative purposes, either for financial planning advice or for designing default programs such as for 401(k) accounts (c.f., Findley and Caliendo, 2008), using a low personal discount rate might be reasonable, as overly discounting the future might lead to future regrets.

CONCLUSIONS

In developing normative analyses for household financial decisions, it is important to start with plausible assumptions about utility function parameters. Assumptions about utility function parameters are also crucial in evaluating whether households are making mistakes in their financial decisions. Assuming a low personal discount rate may be reasonable for normative applications, as some people may have trouble imagining utility they will receive from future consumption. For normative applications, it is plausible to assume that people should discount for risk of death and for planned changes in household size. Using an assumption of intertemporal elasticity equivalent to 5 ($-\varepsilon = 5$) seems reasonable, as that implies a relatively flat consumption path for a wide range of interest rates or rates of return. If only one level of relative risk aversion is to be assumed, a level of 5 or 6 seems reasonable, but it would be better for individual investment advice to use a level more appropriate for an individual. The hypothetical income gambles in the Health and Retirement Study (Barsky et al., 1997) are somewhat complex, and have some limitations in terms of the assumptions respondents might make (c.f., Hanna and Lindamood, 2004). However, Hanna and Lindamood found a significant correlation between responses to their hypothetical pension choice questions and the Survey of Consumer Finances risk tolerance question, which is much easier to present than the income gamble questions.

In the future, normative studies should cite sources that actually estimated utility function parameters, rather than simply arbitrarily asserting a value or citing another study that also arbitrarily asserted a value.

APPENDIX 1

Summary of Each Article in Table I

Hubbard, Skinner, and Zeldes (1994) derived optimal savings levels and examine predictions of a life-cycle simulation model subject to uninsured idiosyncratic risk (uncertainty about earnings, medical expense, and lifespan). For the benchmark case, the relative risk aversion coefficient was assumed to be 3, which is consistent with many empirical studies such as Skinner (1985). The discount rate was assumed to be 3% per year, and no specific reference was given. To examine the sensitivity, they also set the relative risk aversion coefficient to be 1 and 5, and set the discount rate to be 1.5% and 10% per year.

Campbell and Viceira (1999) derived the optimal portfolio allocations and optimal savings levels of an infinitely lived investor who faces a constant riskless interest rate and a time-varying equity premium by employing Epstein Zin Weil utility. In order to compare the portfolio allocations and consumption rules implied by low and high risk aversion coefficients, they use eight levels of relative risk aversion coefficients and elasticity of intertemporal substitution from 0.75 through 40. The discount rate is assumed to be 6% per year. No reference is provided for general discounting.

Brown and Poterba (2000) derived the optimal savings levels of married couples by purchasing joint life annuities. They assume that the household utility function is a weighted sum of the sub-utility functions for the husband and the wife, and each of the sub-utility functions has constant relative risk aversion. Since Barsky et al. (1997) suggested that household risk aversion levels are higher than unity, they consider four different levels of values for risk aversion from 1 through 10. They also assume that both members of the couple have the same discount rate.

Campbell, Cocco, Gomes, Maenhout, and Viceira (2001) derived the optimal portfolio allocations and optimal savings levels of an infinitely-lived investor who faces a time-varying equity premium. They solve the model for five levels of relative risk aversion coefficients and elasticities of intertemporal substitution from 1 through 20. They emphasize on those values because time-series studies of representative-agent models suggest that the elasticity of intertemporal substitution is well above one and may be close to infinity (Hall 1988).

Viceira (2001) derived the optimal portfolio allocations and optimal savings levels for long-horizon investors with non-tradable labor income. The investor's preferences are described by a standard, time separable, power instantaneous utility function over consumption. The expected lifetime after retirement was set to 10 years, and the discount rate was set to 9.1% per year in order to ensure that the investor is impatient but Viceira did not cite a source. He reported the optimal portfolio choices for six levels of relative risk aversion coefficients from 2 through 12.

Gomes and Michaelides (2003) derived the optimal portfolio allocations and optimal savings levels in a life-cycle model with liquidity constraints, undiversifiable labor income risk and stock-market participation costs. For the coefficient of relative risk aversion, the benchmark value was assumed to be 2, and higher values 5 and 8 were also studied. For the discount rate, the benchmark value was assumed to be 5% per year, and higher values 7% and 10% per year were also investigated. They did not cite sources for their parameter assumptions.

Cocco (2005) derived the optimal portfolio allocations with investment in housing. The coefficient of relative risk aversion was set to 5, which is below the upper bound of 10 considered reasonable by Mehra and Prescott (1985). The discount rate was assumed to be 4%

per year. These parameters were assumed to at least roughly match the mean levels of housing and mortgage relative to financial assets observed in the data.

Gomes and Michaelides (2005) derived the optimal portfolio allocations in a life-cycle model with Epstein-Zin preferences, a fixed stock market entry cost and moderate heterogeneity in risk aversion. They first chose the relatively standard choice in preference parameters: risk aversion was set to 5, the elasticity of intertemporal substitution coefficient was set to 5, and the discount rate was set to 4% per year. For sensitivity analysis, they used several different values {1, 1.2, 2, 4, 5} for risk aversion and the elasticity of intertemporal substitution coefficient. They did not provide specific references for the parameter values.

Cocco, Gomes and Maenhout (2005) derived optimal portfolio allocations and optimal savings levels in a life-cycle model with non-tradable labor income and borrowing constraints. The coefficient of relative risk aversion was set to 10, which is the upper bound for risk aversion considered reasonable by Mehra and Prescott (1985). The discount rate was set to 4% per year, plus discounting for risk of death. With Epstein-Zin utility, they also studed the optimal portfolio policies for different values of the elasticity of intertemporal substitution {2, 5, 10 (the benchmark)} and with relative risk aversion equal to 10. No reference was provided for general discounting.

Scholz, Seshadri, and Khitatrakun (2006) derived optimal savings levels using a life cycle model that incorporates uncertain lifetimes, uninsurable earnings and medical expenses, progressive taxation, government transfers, and pensions and social security benefits. The discount rate was set as 4% per year, and the coefficient of relative risk aversion and the elasticity of intertemporal substitution were set to 3. These parameters were similar to those used by Hubbard et al. (1995) and Engen et al. (1999).

Calvet, Campbell, and Sodini (2007) investigated the efficiency of household investment decisions in a unique dataset containing the disaggregated wealth and income of the entire population of Sweden. They computed utility losses for a range of possible risk aversion for a typical investor with a relative Sharpe ratio loss of 35%. They found that the median utility loss roughly coincided with a calibrated value of risk aversion between 4 and 6. They did not cite sources for their parameter assumptions.

Livishits, Macgee and Tertil (2007) analyzed two different consumer arrangements, Chapter 7 of the US bankruptcy code and a system motivated by continental Europe. To evaluate consumer bankruptcy laws quantitatively, they used a heterogeneous agent life cycle model. They set the annual discount rate equal to 6% per year, and the relative risk aversion coefficient was assumed to be 2. No specific reference was given for each parameter.

Gomes and Michaelides (2008) derived optimal portfolio allocations in a life-cycle model with incomplete markets and heterogeneous agents. They considered two groups of agents with the same population size and different preference parameters: type-A agents have low risk aversion (assumed 1.1) and low elasticity of intertemporal substitution (assumed 10), and type-B agents have high risk aversion (assumed 5) and slightly higher elasticity of intertemporal substitution (assumed 2.5). The discount rate was assumed to be 1% per year for both groups. No specific reference was given for the parameter assumptions.

Lopes (2008) conducted empirical analysis of personal bankruptcy filing by using parameterized model of optimal consumption allowing for borrowing and default. For the benchmark case, the relative risk aversion coefficient was assumed to be 3, and the discount rate was assumed to be 3% per year. These are also the benchmark parameters used by Deaton (1991) and Carroll (1997).

Hornef, Maurer and Stamos (2008) derived optimal portfolio allocations and optimal savings levels over the life cycle for households facing uninsurable labor income risk, ruin risk, stochastic capital markets, and uncertain lifetime. The preference parameters were set to standard values found in the life-cycle literature (e.g. Gomes and Michaelides, 2005): the coefficient of relative risk aversion was 5, the elasticity of intertemporal substitution was 5, and the discount rate was 4% per year. For a sensitivity analysis, they used two different values of risk aversion and elasticity of intertemporal substitution coefficient {2, 5}.

Horneff, Maurer, and Stamos (2008d) derived optimal portfolio allocations for a retiree with Epstein-Zin utility. For the base case, the coefficient of relative risk aversion was set to 5, the elasticity of intertemporal substitution was set to 5, and the discount rate was 4% per year. This calibration was consistent with the recent life-cycle literature such as Cocco, Gomes, and Maenhout (2005). For a sensitivity analysis, they used several different values {2, 5, 10} for risk aversion and elasticity of intertemporal substitution coefficient.

Kuznitz, Kandel, and Fos (2008d) derived the optimal portfolio allocations and optimal savings levels of a long-lived investor who gets pleasure not only from current consumption, but also from the contemplation of future consumption. They used six levels of relative risk aversion coefficients from 5 through 30, in order to solve the investor’s problem for different values of relative risk aversion. The subjective discount rate was set to 6% per year as in Campbell and Viceira (1999).

APPENDIX 2

Summary of Each Article with Estimates of Intertemporal Elasticity and of Relative Risk Aversion

Skinner (1985) analyzed the response of consumption to interest rate changes in order to estimate the intertemporal elasticity of substitution between current and future consumption. To estimate the extent to which consumers respond to mortality probabilities in making their consumption decisions, the 1972-1973 Consumer Expenditure Survey was used The intertemporal elasticity calculated from these estimates ranged from 2 to 5, with a midpoint of 3.5.

Hall (1988) estimated the intertemporal elasticity based on the response of the rate of change of consumption to changes in the expected real interest rate. Using inflation and stock price expectations recorded in the Livingston survey, the elasticity estimate ranged from 2.9 to 15. The range estimated from monthly consumption data is from -3.3 to 3.3, and the range estimated from quarterly data is from 2.9 to 10.

Barsky, Juster, Kimball and Shapiro (1997) used experimental questions to estimate the elasticity of intertemporal substitution. The point estimate of the average elasticity of intertemporal elasticity was 5.

Hanna et al. (2003) presented to students a version of alternate consumption paths given a hypothetical real income projection and stipulating that the respondents should assume they could enjoy consumption as much in the future as today (0% personal discount rate), and the household situation would be the same in the future. The interquartile range of the values of elasticity, $-\varepsilon$, were 2.9 to 8.4, with a midpoint 5.6. The midpoint estimate of $-\varepsilon$ was similar to the midpoint estimate of relative risk aversion from hypothetical pension gamble questions, but the two measures were not positively correlated.

Mehra and Prescott (1985) analyzed the equity premium and attempted to find values of relative risk aversion that would be consistent with the much higher return on stocks compared to government bonds in the U.S. The midpoint of the estimate obtained for relative risk aversion was 5.

Barsky, Juster, Kimball and Shapiro (1997) used hypothetical questions about risky jobs to estimate the relative risk aversion is estimated from questions about choices of hypothetical jobs by responses in Health and Retirement Study. The midpoint of the estimated relative risk aversion was 6, but 65% of respondents were in the top category of over 3.8, and 76% had relative risk aversion of 2.0 or higher.

Hanna, Gutter and Fan (2001) estimated the relative risk aversion from choices of hypothetical pensions by a web-based survey of students and general public. The interquartile range of their estimates was from 2 to 8.4, with a midpoint 5.6.

Hanna, Gutter and Fisher (2003) used choices of hypothetical consumption paths by 252 students at two universities to elicit values of intertemporal elasticity. If we consider Equation 5, in order to identify one of these three parameters from actual or hypothetical choices, we need to know the values of the other two parameters. So, for instance, if we know the real interest rate faced by a household and assume a value for the personal discount rate, we can infer the intertemporal elasticity. Hanna et al. (2003) instructed respondents to assume they would have the same capacity to enjoy consumption until death, which would not take place until age 100, so implicitly there was an assumption of a 0% discount rate. An interquartile range of 2.9 to 8.4, with a midpoint of 5.6 for the elasticity was observed. The midpoint for elasticity was similar to the midpoint obtained for relative risk aversion based on hypothetical pensions choices for the same sample of students, but the measures were not significantly correlated.

Attanasio, Banks and Tanner (2002) estimated ownership probabilities to separate shareholders from non-shareholders, estimates of the intertemporal elasticity for different groups were constructed. They used quarterly data on aggregate total expenditure from the UK National Accounts to compute estimates of the Intertemporal Marginal Rate of Substitution. The coefficient of relative risk aversion was assumed between 0.5 and 5, and the discount rate was assumed to 2% per year.

Hanna and Lindamood (2004) used choices of graphical presentation of hypothetical pensions by web sample of students to estimate relative risk aversion. The approximate interquartile range was from 2 to 6, with a mean of 4.4.

Fang and Hanna (2008) used the 2004 HRS job choice measure, and found a mean level of relative risk aversion of 5.8, with 39% having a level over 7.5, and 22% having a level under 3.8.

APPENDIX 3

Summary of Articles with Estimates of the Personal Discount Rate

Lawrance (1991) used Euler equations to estimate household time preferences across different socioeconomic groups. Controlling for age, education, and racial composition, average time preference rates varied from 0.8 % for households in the highest 5% of the labor income distribution to 3.5% for households whose labor income were in the bottom fifth percentile.

Under the buffer-stock model, Carroll and Samwick (1997) developed a methodology for estimating a time preference rate which would be consistent with empirical results using the Panel Study on Income Dynamics (1981-1987). The study reported point estimates for the discount rate ranging from 5% to 14%.

Samwick (1998) estimated the distribution of time preference rates by using different combinations of risk aversion coefficients and initial asset levels based on 1992 Survey of Consumer Finance (SCF) dataset. For the baseline case, the median rate of time preference was 7.6% per year. The interquartile range was 2.9% to 14.7%.

Gourinchas and Parker (2003) estimated time preference parameter by using the dynamic stochastic model of the life-cycle saving behavior of households. Based on the 1980-1993 Consumer Expenditure Survey and 1989-1994 Panel Study on Income Dynamics, the average household had a discount rate of 4.0%-4.5% per year.

Barsky et al. (1997) measured time preference by asking respondents in the 1992 Health and Retirement Study (HRS) to choose consumption profiles implicitly associated with different rates of return. The midpoint of estimates was 0.8% per year, with a lower bound of 0.3% and an upper bound of 1.3%.

Coller and Williams (1999) elicited individual discount rates by using a controlled experimental design involving investment decisions. The median discount rate implied by subject choices for all experimental sessions combined was in the interval of 17.5%–20% per year.

Harrison, Lau and Williams (2002) estimated individual discount rates using European Community Household Panel Survey (ECHP). Their results presented that nominal discount rates did not vary significantly over the time horizon, one-year to three year. However, there were variations in individual discount rates with respect to socio-demographical variables. The average discount rate elicited over all subjects was 28%.

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Table I
Normative Studies

Study	Intertemporal elasticity (expressed as relative risk aversion)	Relative risk aversion	Discounting of utility of future consumption	Focus, other notes
Hubbard, Skinner, and Zeldes, 1994; 1995	Benchmark 3 (Skinner, 1985) Tried 1 and 5	Benchmark 3 (Skinner, 1985) Tried 1 and 5	Benchmark 3% per year (No specific reference) Tried 1.5% and 10% per year	OS, considering a life-cycle model of consumption, saving, and wealth accumulation subject to uninsured idiosyncratic risk.
Campbell and Viceira, 1999	4 (Hall, 1988) Tried 0.75-40	4 (Hall, 1988) Tried 0.75-40	6% per year (No specific reference)	OP; OS, assuming an infinitely lived investor with Epstein-Zin utility who faces a constant riskless interest rate and a time-varying equity premium.
Brown and Poterba, 2000	N/A	5(Mehra and Prescott, 1985; Barsky, et al., 1997) Tried 1-10	mortality rate at age 20, rate= 0.1% at age 72, rate=1% at age 80, rate=5%	Optimal annuity choice. Assume the household utility function is a weighted sum of the sub-utility functions for the husband and the wife.
Campbell, Cocco, Gomes, Maenhout, and Viceira, 2001	4 (Hall, 1988) Tried 1-20	4 (Hall, 1988) Tried 1-20	6% per year (No specific reference)	OP; OS, assuming an infinitely-lived investor who faces a time-varying equity premium.
Viceira, 2001	3 (Campbell and Viceira, 1999) Tried 2-12	3 (Campbell and Viceira, 1999) Tried 2-12	9.1% per year, the expected lifetime after retirement is set to 10 years.	OP; OS, assuming long-horizon investors with non-tradable labor income.
Gomes and Michaelides, 2003	2 (Campbell, Cocco, Gomes, and Maenhout, 2001) Tried 2-8	2 (Campbell, Cocco, Gomes, and Maenhout, 2001) Tried 2-8	5% per year. Mortality tables of the National Center for Health Statistics used for survival probabilities. Tried 5-10%	OP; OS, assuming a model with uninsurable labor income risk and stock market participation costs.
Cocco, 2005	N/A	5 (below the upper bound of 10 considered plausible by Mehra and Prescott,1985)	4% per year (No specific reference)	OP with investment in housing, assuming that the investor derives utility from both housing and nondurable goods.
Cocco, Gomes, and Maenhout, 2005	10 (Mehra and Prescott, 1985) Tried 2-10	10 (Mehra and Prescott, 1985) Tried 2-10	4% per year, plus discounting for risk of death. (No specific reference). Mortality tables of the National Center for Health Statistics used for survival probabilities	OS; OP
Gomes and Michaelides, 2005	5 (Attanasio, Banks and Tanner, 2002) Tried 1-5	5 (Attanasio, Banks and Tanner, 2002) Tried 1-5	4% per year (No specific reference). Mortality tables of the National Center for Health Statistics used for survival probabilities	OP. Epstein–Zin preferences, Assuming Real interest rate=2%, Equity premium=4%, Stock volatility=18%
Scholz, Seshadri, and Khitatrakun, 2006	3 (Hubbard et al. , 1995 and Engen et al. 1999)	3 (Hubbard et al. , 1995 ; Engen et al. 1999)	4% per year, (3% in Hubbard et al. , 1995 and Engen et al. 1999) Survival Probabilities are	Assess whether households are saving optimally for retirement.

Study	Intertemporal elasticity (expressed as relative risk aversion)	Relative risk aversion	Discounting of utility of future consumption	Focus, other notes
			based on the 2002 life tables of the Centers for Disease Control and Prevention, U.S. Department of Health and Human Services	
Livshits, Macgee and Tertil, 2007	2 (No specific reference)	2 (No specific reference)	6% per year (No specific reference)	The quantitative analysis of two different consumer bankruptcy laws by using a heterogeneous agent life cycle model.
Calvet, Campbell, and Sodini, 2007	N/A	5 (No specific reference) Tried 2-7	N/A	OP
Gomes and Michaelides, 2008	Baseline 2.5, 10; Tried 1.3-20 (No specific reference)	Type-A: baseline 1.1, tried 1.05-4 Type-B: Baseline 5, tried 3-5 (No specific reference)	1% per year - No specific reference given. We use the mortality tables of the National Center for Health Statistics to parameterize the conditional survival probabilities.	OP
Hornef, Maurer, and Stamos, 2008	5 (Gomes and Michaelides, 2005) Tried 2-5	5 (Gomes and Michaelides, 2005) Tried 2-5	4% per year (Gomes and Michaelides, 2005)	OP; OS, assuming a dynamic utility maximizer with CRRA and Epstein/Zin preferences who has access to liquid stocks, bonds, and illiquid life annuities.
Kuznitz, Kandel, and Fos, 2008	N/A	15 (Campbell and Viceira, 1999, 2000) Tried 5-30	6% per year (Campbell and Viceira, 1999)	OP; OS, assuming a long-lived consumer who faces a time-varying equity premium
Lopes, 2008	3 (Deaton 1991 and Carroll 1997)	3 (Deaton 1991; Carroll 1997)	3% per year (Deaton, 1991; Carroll 1997)	Analysis of personal bankruptcy filing by using parameterized model of optimal consumption
Horneff, Maurer, and Stamos, 2008	5 (Cocco, Gomes, and Maenhout, 2005) Tried 2-10	5 (Cocco, Gomes, and Maenhout, 2005) Tried 2-10	4% per year (Cocco, Gomes, and Maenhout, 2005). Survival probabilities for pricing the annuity are also taken from the 2000 Population Basic mortality table	OP Epstein–Zin preferences, Assuming Real interest rate=2%, Equity premium=4%, Stock volatility=18%

OS= derive optimal savings levels.
OP= derive optimal portfolio allocations

Table II
Estimates of Intertemporal Elasticity

Study	Midpoint	Notes
Skinner, 1985	3	Empirical estimate of the intertemporal elasticity of substitution between current and future consumption, ranged from 2 to 5.
Hall, 1988	6	Empirical analysis of the elasticity of intertemporal substitution; range estimated from Livington Survey=2.9-15, range estimated from monthly data=(-3.3)-3.3, range estimated from Postwar Quarterly Data=2.9-10
Barsky, et al., 1997	5	Use designed experimental questions in the Health and Retirement Study to estimate the elasticity of intertemporal substitution.
Hanna, et al., 2003	5.6	Choices of hypothetical consumption paths by student sample. Interquartile range=2.9-8.4

Table III
Estimates of Relative Risk Aversion

Study	Midpoint or Mean	Notes
Mehra and Prescott, 1985	5	Empirical analysis of equity premium
Barsky, et al., 1997	6	Choices of hypothetical jobs by older Americans in Health and Retirement Study (HRS), but 65% of respondents were in the top category of over 3.8, and 76% had relative risk aversion of 2.0 or higher.
Hanna, et al., 2001	5.6	Choices of hypothetical pensions by web sample of students and general public. Interquartile range=2-8.4
Attanasio, Banks and Tanner, 2002	3	Empirical analysis of equity premium; show that the risk aversion coefficient is much higher for non-stockholders than for stockholders.
Hanna and Lindamood, 2004	4.4	Choices of graphical presentation of hypothetical pensions by web sample of students Interquartile range=2-6
Fang and Hanna, 2008	5.8	Used the 2004 HRS job choice measure, with 39% having level > 7.5, 22% having a level <3.8

Table IV
Estimates of Annual Personal Discount Rate

Study	Midpoint	Notes
Lawrance, 1991	3.5% (5th percentile) 0.8% (95th percentile)	Optimal Saving PSID (1974-1982) No liquidity constraint Interest rate: treasury bill rate and passbook saving rate
Barsky, et al., 1997	0.78% (Midpoint) 0.28% (Lower bound) 1.28% (Upper bound)	Experimental study (1992 HRS)
Carroll and Samwick, 1997	Range from 5% to 14%	Optimal Saving PSID (1981-1987) No liquidity constraint Interest rate: 0% (baseline) and 3% (HSZ model)
Samwick, 1998	7.63% (Median) 2.93% (25th percentile) 14.66% (75th percentile)	Optimal Saving (1992 SCF) Real interest rate: 4% (after tax return to portfolio)
Coller and Williams, 1999	17.5-20% (Median)	Experimental study (Graduate and undergraduate students majoring in business)
Harrison, Lau and Williams, 2002	28% (Average)	Experimental study (Nationally representative sample in Denmark)
Gourinchas and Parker, 2003	4.0-4.5% (Average)	Optimal Saving CES (1980-1993) and PSID(1989-1994) Interest rate: 3.440 (average real interest rate from January 1980 to March 1993)

PSID = Panel Study of Income Dynamics
HRS = Health and Retirement Study
SCF = Survey of Consumer Finance
CES = Consumer Expenditure Survey