# Risks and Opportunities of Inverse ETFs for Long Term Investors

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## Abstract

The continued growth of ETFs presents additional opportunities for individual investors as well as accounts managed by financial services professionals. Certain new ETF products, such as inverse and leveraged ETFs, are not generally recommended for long-term passive investment strategies which many individual investors and advisors follow. This paper will research the long-term impact of portfolio performance using passive investment strategies that employ inverse ETFs in their portfolios. The assessment will be accomplished through the use of Monte Carlo simulation. The results of the study quantify the benefits and detriments in the use of these new ETF products, and may materially alter long-term passive portfolio construction.


## 1. Introduction

The recent economic downturn has called into question the effectiveness of diversification and led to some investors to consider alternative investments as part of their overall portfolio. A recent study by Arshanapalli et al (2010) shows that, while more severe than many of the bear markets before it, the recent downturn did benefit from diversification and alternative investments such as gold. So-called "wide diversification" is not entirely new (See Mulvey et al 2007 or Gibson 2004), and suggests that increasing the number of asset classes offers significant benefit. But, the use of inverse products as a new asset class in a diversified portfolio has yet to be examined.

Lack of study at a portfolio level does not suggest little is known about the impact of individuals holding inverse ETFs. In fact, to the contrary, a comprehensive study by Cheng and Madhaven (2009) shows how inverse ETFs need to be re-balanced on a daily basis in order to maintain a constant leverage, and how this can lead to wealth destruction. This wealth destruction occurs largely because of the resulting path dependence on accumulated wealth that can easily diverge from the underlying index over longer holding periods. This wealth destruction is also aggravated by higher volatility, although Trainor (2008) notes an interesting corollary that suggests some of these leveraged ETFs can outperform their respective benchmarks in periods of low volatility.

### *1.1. Background of Inverse ETFs*

While the first inverse ETF came into existence in 2007, both inverse and leveraged open-ended mutual funds have been in existence far longer. Since 1997, Profunds has offered an inverse and leveraged version of the S&P 500 index through two mutual funds. By mid 2010, 150 different inverse and leveraged ETFs were available with a total of $30B of assets under management. (See Guedj et al 2010) Although their track record is brief, the history of these products so far suggests that they are meeting the

objectives contained within their offering prospectuses. Specifically for this proposed research paper, they appear to effectively meet their daily objectives of inverse daily return of their underlying index through the use of swaps and other derivative instruments. Unfortunately, the daily re-leveraging of these products resulted in potential investment management challenges and risks.[1] Many researchers, including Barnhorst and Cocozza (2010), Cheng and Madhaven (2009), Guedj et al (2010), and Trainor and Baryla (2008) have noted that daily re-balacing has tended to reduce their stated investment effectiveness over longer holding periods. Furthermore, the limited history of these products prevents extensive application of historical data within a diversified portfolio.

This research paper addresses the effect of a buy-and-hold portfolio containing inverse ETFs and their investment effectiveness as part of a diversified portfolio. We conduct a Monte Carlo simulation over longer time horizons than is historically available for inverse ETFs. The objective is to determine whether there is any risk-return benefit of these products in a diversified portfolio, or whether, as the previous studies have shown, they should be avoided entirely by buy-and-hold investors. Perhaps, these products should remain clearly in the hand of speculators and day traders. Greater regulatory protections for retail investors may be warranted.

*1.2 Regulation of ETFs*

ETFs, in general, have not had any unique regulations imposed on them. The current environment seems directed towards increased specific regulation. The push for this regulation was initiated by criticisms of leveraged and inverse leveraged products, such as those under review in this paper. The best way to address the regulatory issue of ETFs is not so much of a literature search but to focus on a very recent announcement made the week of October 3, 2011 by Blackrock Inc[2]. In a report by them, they addressed the growing concerns that investors do not fully understand ETFs or the risks and costs associated with them. Blackrock therefore called for greater transparency and uniform regulation. (Harman 2011)

Blackrock noted five items that need to be addressed on a uniform basis by all providers of ETFs. These are as follows:

1. Clear labeling of product structure and investment objectives.
2. Frequent and timely disclosure of holdings and exposures
3. Clear standards for diversifying counterparties and quality of collateral.
4. Disclosure of all fees and costs paid including those to counterparties.
5. Universal trade reporting for all equity trades, including ETFs.

[1] Fidelity at http://personal.fidelity.com/research/etf/content/leveraged_etn_etf.shtml noted that because of rebalancing and other risks, leverage and inverse leveraged ETFs are intended as short term trading vehicles for sophistical investors actively monitoring their portfolios on a daily basis.

[2] Blackrock owns iShares, is currently the world's largest provider of ETF products.

It would appear that more regulation of ETFs will be forthcoming due not only to requests by investors and regulators but now by the providers of ETFs themselves. In fact, industry experts will be among those testifying at the U.S. Senate Banking sub-committee hearing in October 2011, entitled "Market Microstructure: Examination of Exchange-Traded Funds (ETFs)."

## 2. Research Hypothesis

This research paper attempts to uncover whether there are any potential diversification benefits to a portfolio that holds a portion of its positions in inverse ETFs. To test this hypothesis, we examined three typical long-term buy-and-hold investors who would typically seek some diversification by holding a broad stock and bond index fund in a tax deferred retirement account. We assume such investors would be representative of three separate tolerances to risk, and that the risk tolerance can be adjusted based on their portfolio exposure to stocks and bonds.

### *2.1 Strategic Allocation Between Stocks and Bonds*

The strategic allocation between stocks and bonds is highly dependent on the time horizon of the investor and the tolerance for risk. This sounds like an easy task, but it clearly is not in practice. Capital Market Theory clearly states that there is no optimal point on the Efficient Frontier, the Capital Market Line or the Security Market Line. But to be optimal, one must be on the efficient frontier or the line.

Thusly, the Capital Market Theory does not give us a practical strategic allocation. Friedman and Savage (1948) suggested that an important class of risk can be rationalized by a rather simple extension of orthodox utility analysis. This becomes far more useful with Markowitz's Efficient Frontier construct (1952). Using one's own utility function, one could optimally place themselves on that frontier with its classical tradeoff between risk and return. Henry Latane (1959) stated it correctly when he noted that Markowitz's analysis does not give any basis for choice among the efficient portfolios except the individual's safety preference rate developed through whatever construct he or she wishes. So, even if one incorporates Modigliani and Brumberg's life cycle theory of savings (1954); Latane's criteria for choice among risky assets (1959); Thaler and Shefin's inquiry into economic theory and self-control (1981); Tversky and Kaherman's loss aversion in riskless choices (1991); or Herbert A. Simon's inquiry into a behavioral model of rational choices (1995), one is still left without a workable pragmatic allocation system.

This failure to quantify risk in an operational context for the strategic allocation of investment assets has caused the rise of mechanical methods that reflects classical thinking by connecting risk-aversion to one's age. Thusly, the "Rule of 120" has appeared in the literature. This suggests that for the stock allocation should be 120 minus the age of the individual. The rule has been modified by others from about 100 to 130 changing due to different economic and investment cycles. The remainder of the money

is then allocated to fixed interest instruments[3]. The popularity of this approach has motivated many fund providers to offer single funds that automatically follow this approach, naming them either Lifecycle or Target Date Funds (TDFs).

Consequently, this approach was undertaken for this paper to represent different levels of risk aversion. Recognizing that the time period selected might have been impacted by the economic and investment climate, we chose to examine TDFs at a time period at the end of 2006 before the economic financial meltdown. Table 1 was obtained from the bargaineering blog, showing the relationship of selected lifecycle funds of Vanguard, Fidelity, and T. Rowe Price. The average of 88% in Table 1 for the longest time horizon TDFs is line with other studies. Poterba (2009) reported 88% in March 2005. This is to be followed by Pang and Waeskawsky (2010), which reported 86% in May 2008.

**Table 1**
Stock allocation used in by leading firms, circa 2007

| Age | Rule of 120 | Vanguard | Fidelity | T. Rowe Price | Average | Difference from Rule of 120 |
|---|---|---|---|---|---|---|
| **56** | 64% | 65.16% | 57.65% | 70.41% | 64% | 0 |
| **41** | 79% | 83.82% | 80.98% | 87.64% | 84% | +5% |
| **26** | 94% | 88.66% | 86.93% | 88.86% | 88% | -6% |

This table shows the reconciliation of the "Rule of 120" against leading financial services firms, as posted in 2007 on http://www.bargaineering.com/articles/stock-allocation-rule-120-minus-age.html

It should be noted that the perceived likelihood of adjustments to TDFs has in fact occurred. Pang and Waeshawsky (2011) have noted that TDFs are deducting the risky equity exposure from past levels for investors near retirement. Their simulation results show that glide path designs are important determinates of wealth levels and volatilities. They further stated that as a sole vehicle for retirement wealth accumulation must be considered risky especially when the possibilities of large economic disasters are considered. The latter is of particular importance to the research hypothesis being investigated in this paper. Nevertheless, based on the above discussion, Table 2 was formatted to reflect the strategic asset allocation percentages used in this study. We further assume that the investor is does not re-balance, but rather executes a buy-and-hold at the beginning of a 10 year investment period.

**Table 2**
Traditional stock and bond allocation

| Risk Tolerance | High | Medium | Low |
|---|---|---|---|
| **Typical Age** | 25 | 40 | 55 |
| **Bond Allocation** | 5% | 20% | 35% |
| **Stock Allocation** | 95% | 80% | 65% |

[3] The US Securities and Exchange Commission (2011), while not stating the Rule of 120, implied it indirectly in their suggestion to for investors to consider Target Date Funds (TDF).

| | | | |
|---|---|---|---|
| **Stock:Bond ratio** | 19:1 | 4:1 | 13:7 (1.86:1) |

This table shows the traditional stock and bond initial allocations assumed for simulated 10-year holding period returns.

In contrast, we are interested in comparing simulated terminal wealth of the above investors versus ones who choose to allocate a small portion of their assets at the beginning of the 10-year holding period to an inverse stock ETF. This alternative group of investors is represented by allocations that appear in Table 3, and includes the same stock to bond ratio as established in Table 2. The distinction in Table 3 is that 10% of the investors retirement account will be allocated towards an inverse stock fund. Again, we assume that the investor is does not re-balance, but rather executes a buy-and-hold at the beginning of a 10 year investment period.

**Table 3**
Alternative stock, bond, and inverse stock allocation

| Risk Tolerance | High | Medium | Low |
|---|---|---|---|
| **Typical Age** | 25 | 40 | 55 |
| **Bond Allocation** | 4.5% | 18.0% | 31.5% |
| **Stock Allocation** | 85.5% | 72.0% | 58.5% |
| **Inverse Stock Allocation** | 10.0% | 10.0% | 10% |
| **Stock:Bond ratio** | 19:1 | 4:1 | 13:7 (1.86:1) |

This table shows the alternative stock, bond, and inverse stock initial allocations assumed for simulated 10-year holding period returns.

## 3. Research Methods

We have previously noted the logic for the strategic asset allocation between stocks and bonds. While many believe that one must at the same time be specific about the types of equity or fixed income asset chosen, we believe that this is more of a tactical asset allocation question. For example, equities could be small cap; mid cap, large cap, or REITs. For fixed income, this could be short-term bonds; intermediate-term bonds; long-term bonds, high yield bonds; inflation-indexed; or money markets and/or T-bills. Further complicating the issue, one could consider for equities international exposure and for bonds the question of corporate vs. government bonds further complicated by domestic vs. international. Thusly, the possible pragmatic tactical choices are large.

### *3.1. Proxy selection*

To minimize potential problems we believe arise from the tactical allocation question, we have chosen two well-known and large mutual funds highly representative of rational investment choices as proxies for stocks and bonds. The Vanguard S&P 500 Index Fund (VFINX) was chosen as the proxy to represent domestic equity. This fund attempts to track the performance of the S&P 500. The S&P 500 is a widely recognized benchmark of the US stock market. This index is a large capitalization almost equally weighted between value and growth equities. The Vanguard Total Bond Market Index Fund (VBMFX) was chosen as the proxy to represent fixed income. This fund attempts to

track the Barclays Capital US Aggregate Float Adjusted index. This index is classified as an intermediate-term domestic index. Both funds are no-load and low-fee as well as being passively administrated. The funds are highly popular with Vanguard investors. The S&P fund is second in size (after the Vanguard Total Stock Market (VTSMX)) while the bond fund is third. Vanguard is further known to have very low benchmark tracking errors thereby making these funds valid proxies.

The inverse stock fund returns were modeled here from the daily returns of the stock returns, multiplied by negative one. We did not attempt to model any divergence from the inverse benchmark, when such a fund may trade at either a discount or premium. But, a recent study of inverse fund performance suggests that such a premium or discount remain fairly small, and quickly revert back to the inverse benchmark as modeled here. Gerasimos (2011) found that in particular, emphasis is given to the ability of these ETFs to meet their daily investment target. In this respect, an average deviation from the daily target amounting to -0.034% is computed. Applying a classification to the deviation from the daily return goal, they found that for about 62% of the examined trading period's duration the return of the average short ETF abstains from its target a maximum rate of 0.5%, either below or above the target.

We don't suggest that our model of inverse fund returns is without error, but do believe it is an appropriate starting point for this investigation. In fact, investors are exposed to other risks due to the construction of the inverse funds. Inverse funds replace equity shares with futures and swaps in order to guarantee the applicable multiples of return. Futures have the benefit of having a clearing corporation stated as the counterparty which is a very favorable credit risk advantage. On the other hand, swaps clear through banks; this adds another element of credit risk. This is a major area of concern not fully understood. In addition, futures also require standard amounts and fixed times to expiration as well as being marked to the market. Swaps do not. Instead, they are more flexible which accounts for their popularity. Choi and Elston (2009) reported that Proshares Inverse S&P 500 ETF held weightings of 91% in swaps and but 9% in futures. Further, Cheng and Madhavan (2009) noted that daily return streams from paired leveraged and inversed leveraged ETFs do not net out on a daily basis.

*3.2. Time period selections*

Due to the recent development of inverse ETFs, and uncertainty of future markets, we developed our simulation by drawing from actual frequency distributions observed within the past 20 years. Each frequency distribution was obtained from three recent, but different time periods. We categorized these time periods in terms of overall investor returns as a "most recent case" from 2001 to 2010, a "pessimistic case" from 1999 to 2008, and an "optimistic case" from 1991 to 2000. The descriptive statistics over these periods are summarized in Table 3. Annualized values were determined by assuming an average of 20 trading days in a month. Then, annualized returns are found from the average daily returns multiplied by 20*12 and annualized standard deviations were found from daily standard deviations multiplied by $(20*12)^{1/2}$.

**Table 4**
Return statistics from daily returns over three 10-year periods

| | Return (annualized) | Std Dev (annualized) | Total Return |
|---|---|---|---|
| **Bonds** | 5.22% | 4.39% | 72.5% |
| **Stock** | 3.51% | 21.3% | 13.9% |
| **Inverse Stock** | -3.51% | 21.3% | -45.5% |
| "Most Recent" - January 2nd, 2001 to December 31st, 2010, N=2516 samples | | | |
| **Bonds** | 5.14% | 4.41% | 69.3% |
| **Stock** | 0.74% | 20.7% | -13.7% |
| **Inverse Stock** | -0.74 | 20.7% | -26.2% |
| "Pessimistic" - January 4th, 1999 to December 31st, 2008, N=2516 samples | | | |
| **Bonds** | 7.30% | 4.49% | 113.33% |
| **Stock** | 16.27% | 14.57% | 395.5% |
| **Inverse Stock** | -16.27% | 14.57% | -83.88% |
| "Optimistic" - January 2nd, 1991 to December 29th, 2000, N=2527 samples | | | |

This table shows the return statistics from the three 10-year periods observed from adjusted closing prices of stock and bond market proxies. Inverse stock returns are generated assuming perfect replication of negative daily return from the stock return.

A review of Table 3 suggests several important behaviors were demonstrated in the three intervals considered. First, bond returns were always positive in a fairly narrows annualized return range of 5-7%. Bond volatility was also nearly constant, as measured by the annualized standard deviation ranging between 4.39% - 4.49%. The bond returns were in stark contrast to the returns exhibited by stocks over these three periods. Annualized stock returns varied significantly, from as low as 0.74% to 16.27%. Unlike bonds, there was also a period (1999-2008) when the total return of stocks was negative. This period, as well as the most recent period from 2001-2010, also had a noticeable increase in volatility of approximately 6% (annualized). Lastly, the inverse stock investment generated a total return that was negative in all cases, which is consistent with the wealth destruction expected analytically from Cheng and Madhavan (2009).

An important aspect of Table 3 is that, while the annualized returns are for inverse stocks will always be "negative" of the stock returns, the total returns differ. This is because of the path dependency discussed previously. For the historical periods considered here, the time series of cumulative returns based on a buy-and-hold strategy

are illustrated in Fig. 1 for January 2001 – December 2010, Fig. 2 for January 1999 – December 2008, and Fig. 3 for January 1991 – December 2000.

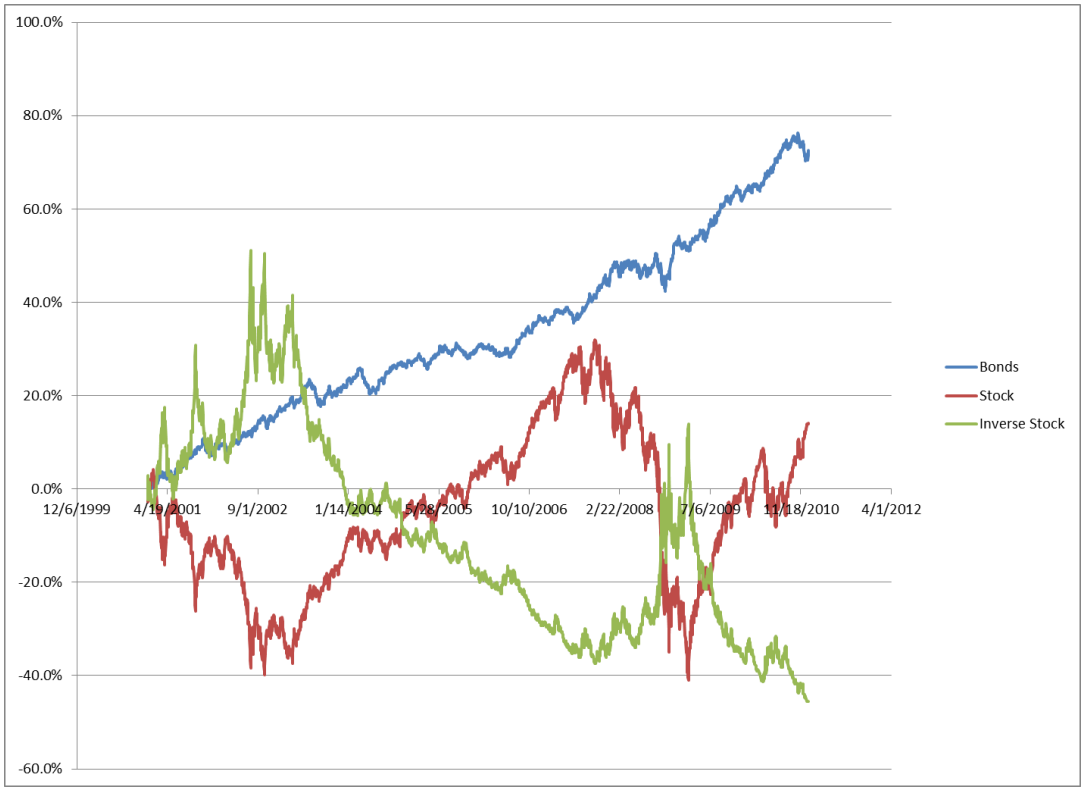



**Fig. 1.** Cumulative return of bonds, stock and inverse stock funds from January 2001 – December 2010

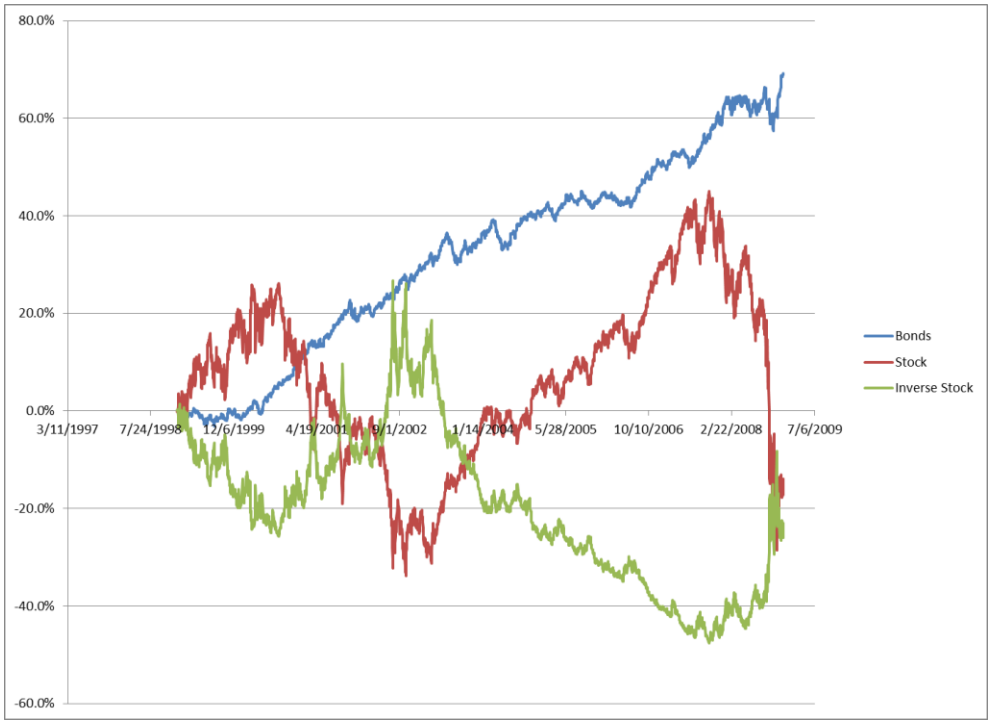



**Fig. 2.** Cumulative return of bonds, stock and inverse stock funds from January 1999 – December 2008

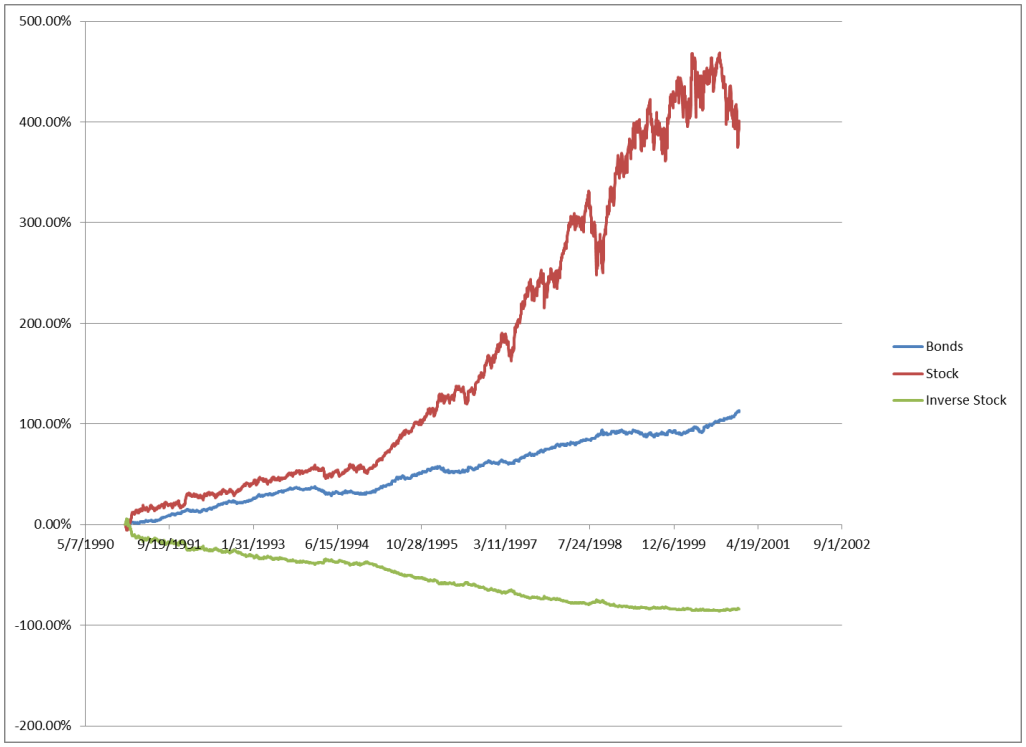



**Fig. 3.** Cumulative return of bonds, stock and inverse stock funds from January 1991 – December 2000

*3.3 Economic and Financial Environment*

The most recent economic and financial period under review in this paper is one of the most turbulent periods in modern history. This should be clearly noted when discussing the findings of this paper. There are numerous ways to discuss this. However, only two have been chosen. These are the stock market highs and lows as noted by Value Line and the ranges of the Barron's Bond Confidence Index.

Value Line notes four relevant periods for this study. The first is their conclusion that a stock market high was achieved on September 4, 1987 followed by a stock market low on October 9, 2002. This was followed by a new stock market high on July 14, 2007 followed by a new stock market low on March 9, 2009. Consequently, major market moves occurred during the study period.

The Barron's Bond Confidence Index is well known and often used by investment professionals in understanding as well as predicting stock market movements. It is found by dividing Barron's High-Grade Bond Index by Barron's Intermediate-Grade Bond Index. A rating of 100 signals high confidence in the economic and financial environment as there is little likelihood of bond default given there is no premium to the lower grade bonds. The index has ranged from a low of about 40 (in the summer of 1940) to about 97 multiple times subsequently. This index widely varied during the study period. The index reached almost 95 in the period 1999-2000 and then fell to about 65 at the end of 2002 and early 2003. The index then rose to above 90 in 2004 and was above 85 just prior to the economic and financial meltdown. It then fell as a result to a modern low in the mid 40s in late 2008 and early 2009. It subsequently moved upwards, and it achieved almost 85 at the end of 2010. It stands at 70.7 on October 17, 2011. A 20-year time series, corresponding to the 20 years considered in this study, appears in Fig. 4.

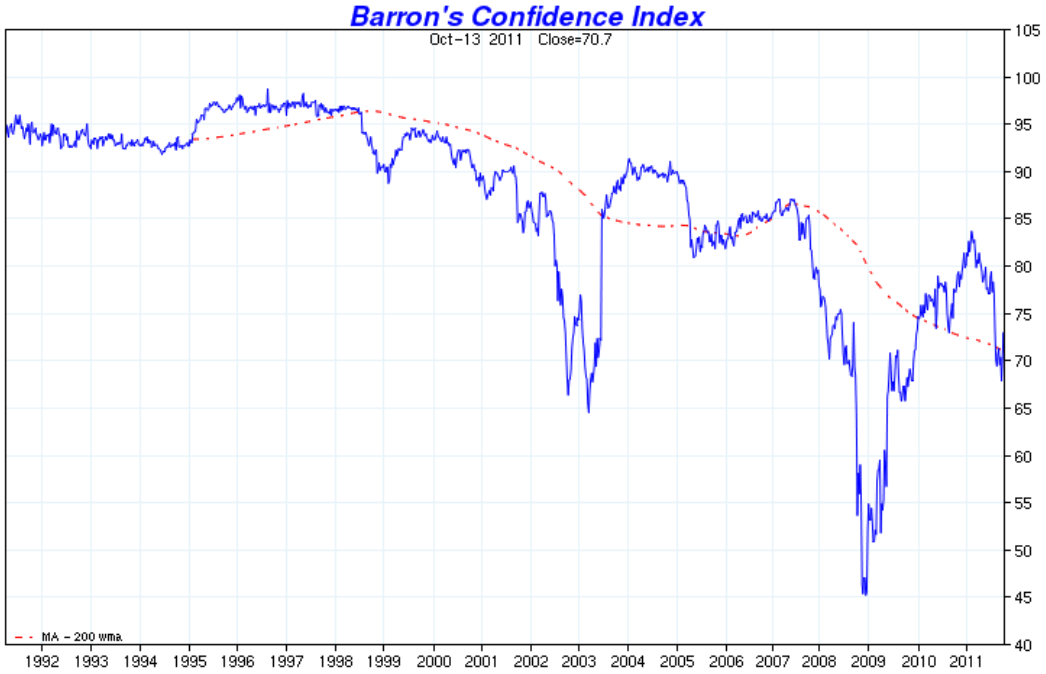



**Fig. 4.** Barron's confidence index (http://www.sharelynx.com/chartstemp/BarronsCI.php)

*3.4 Simulation Methodology*

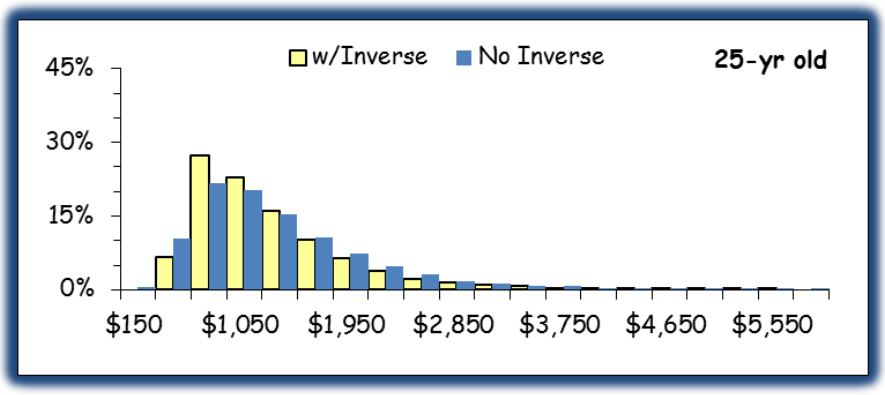
To proceed with a simulation of a portfolio of bonds, stocks and inverse stock funds, we considered two alternatives to simulate future returns. The first was to take any of the three sets of historical returns, fit an appropriate distribution to it, and use this distribution to generate random samples from it. We chose not to follow this method, because of concern about the behavior at the tails of the distribution not reflecting observed returns. Instead, we chose an alternate approach that considered a 20-day consecutive return that was available from the observed return history. This method operates as follows:

1. Assign a numerical index of 1 to N for each of the daily returns observed in the historical 10-year period.
2. For each day, determine the cumulative return up to and including the previous 20 days, so that the accumulated returns from these days are representative of the distribution of monthly returns.
3. Generate 120 random numbers ranging from 1 to N
4. Select returns from the distribution of monthly returns determined in step 2 using the random numbers found in step 3, and generate 10 years of monthly returns.
5. Repeat for 10,000 trials, and determine the mean, standard deviation, and coefficient of variation in terminal wealth.

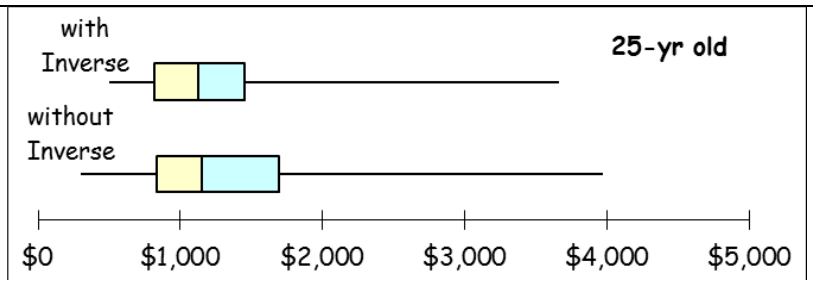
## 4. Simulated Results

*4.1 Simulated returns using "Most Recent" 2001-2010 return distribution*

The distributions of terminal wealth for the three investors classified previously are shown below in Fig. 5, 6 and 7. Within each figure, two distributions are shown, corresponding to the investor with and without a 10% exposure to the inverse stock fund. Each figure also contains a boxplot, indicating minimum, Q1, median, Q3 and maximum values obtained for terminal wealth from the 10,000 trial simulation.

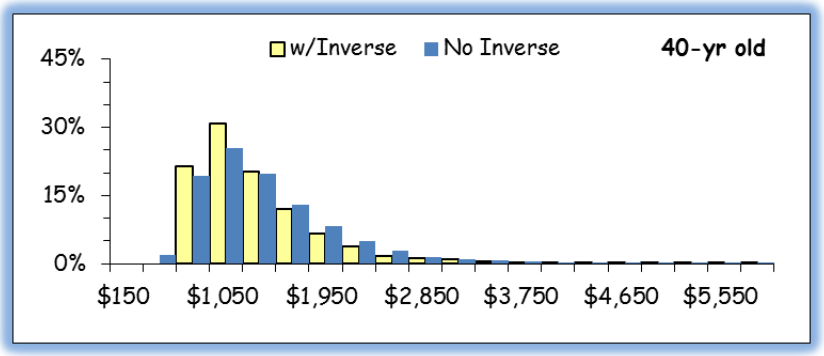



**Panel A: Relative frequency of terminal wealth**

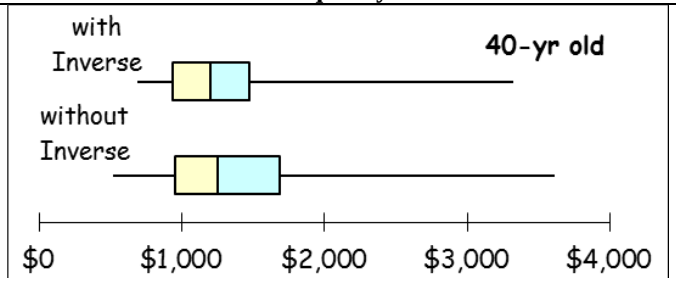



**Panel B: Box plot of terminal wealth**

**Fig. 5.** Terminal wealth distributions for high risk tolerant investor with and without an inverse stock allocation using 2001-2010 return distributions. Distribution assumes starting with $1000, holding investment for 10 years, and negligible transactions costs. Min, Q1, Median, Q3 and Max represented in each boxplot.

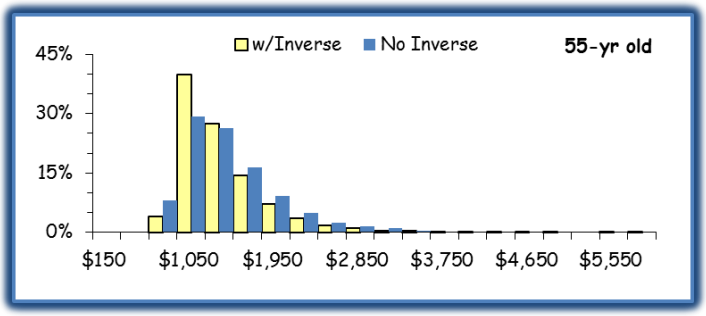



**Panel A: Relative frequency of terminal wealth**

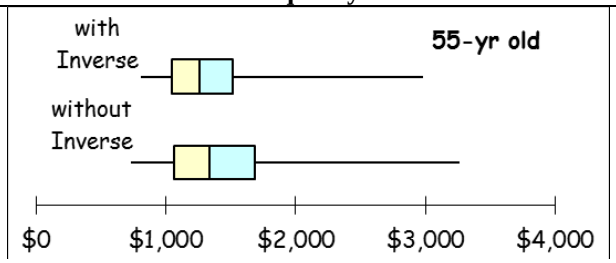



**Panel B: Box plot of terminal wealth**

**Fig. 6.** Terminal wealth distributions for moderate risk tolerant investor with and without an inverse stock allocation using 2001-2010 return distributions. Distribution assumes starting with $1000, holding investment for 10 years, and negligible transactions costs. Min, Q1, Median, Q3 and Max represented in each boxplot.




**Panel A: Relative frequency of terminal wealth**




**Panel B: Box plot of terminal wealth**

**Fig. 7.** Terminal wealth distributions for low risk tolerant investor with and without an inverse stock allocation using 2001-2010 return distributions. Distribution assumes starting with $1000, holding investment for 10 years, and negligible transactions costs. Min, Q1, Median, Q3 and Max represented in each boxplot.

Referring the results of Panel A in Figs. 5-7, it appears that for all investors, the distribution becomes more positively skewed with the addition of the inverse stock fund. The dispersions of the distributions also appear to decrease marginally. These two statistical observations translate to what is expected from knowledge of how the inverse stock fund behaves as a hedge against the stock fund. Thus, the average return is decreased and the variance of the return is also decreased. The reduced dispersion is also evident in Panel B in Figs. 5-7, where the first and third quartiles are closer to one another when the inverse fund is included. Additionally, the range in the terminal wealth values simulated is smaller when the inverse fund is added.

The results from Figs. 5-7 are also encouraging because they show that the inclusion of the inverse fund appears to be simultaneously reducing risk and return. The percentage reduction in mean and standard deviation of terminal wealth are listed in Table 5, along with the corresponding coefficient of variation.

**Table 5**

Reduction in mean and standard deviation of terminal wealth using 2001-2010 return distributions for simulation sampling

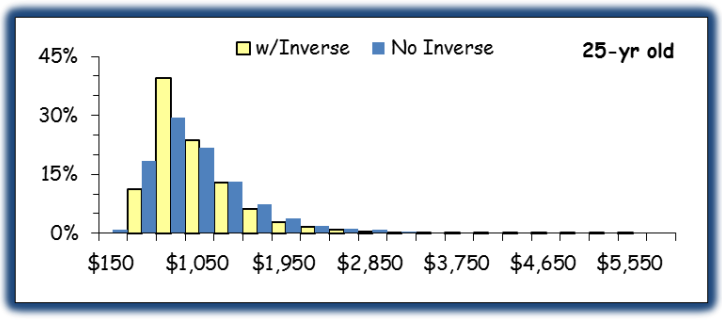
| Terminal Wealth Statistic | 25-yr old | 40 yr-old | 55-yr old |
|---|---|---|---|
| **Standard Deviation** | -13.8% | -14.4% | -15.3% |
| **Mean** | -5.1% | -5.3% | -5.4% |
| **Coefficient of Variation** | -9.2% | -9.7% | -10.5% |

This table shows that the reduction in mean and standard deviation in terminal wealth for each of the three investors identified. Reductions are from 10,000 trials from samples generated from 2001 – 2010.

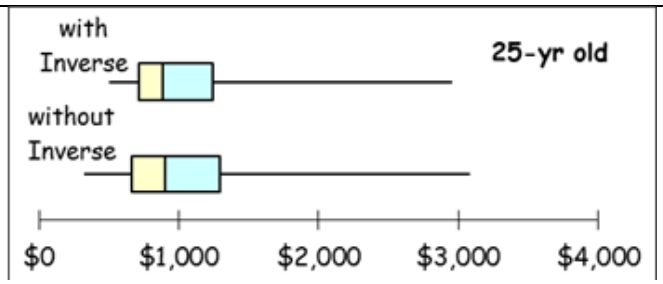
The results in Table 5 are also encouraging for investors seeking improved diversification, suggesting that while returns are reduced, risk as measured by the standard deviation, is reduced to a greater degree. This is evident in the last row of Table 5, which shows the coefficient of variation reduced for each of the three investors considered. The coefficient of variation (CV) is a method to compare assets with different returns. Usually the standard deviation of the asset with the largest return is greater simply because the return is greater. To compare securities with different returns, the CV standardizes (or scales) the standard deviation to make assets with different returns comparable. The CV is a standardized mean-variance tool with the numerator the standard deviation and the denominator the mean return. Consequently, a rational investor prefers a smaller but positive CV.

*4.2 Simulated returns using "Pessimistic" 1999-2008 return distribution*

The results the for a "pessimistic" case shown in Figs. 8-10 and Table 6 are similar to the results found from simulating returns from the "most recent" time interval. Not surprising, the center of the return distribution of total wealth is still reduced. But, the dispersion of total wealth is reduced even more than in our previous "Most Recent" case shown in previously. Table 6 quantifies the greater reduction in dispersion of total wealth, as measured by the standard deviation, providing a further reduction to the coefficient of variation. That is, the CV was previously reduced by approximately 9-10% for the "most recent" case, where now it is reduced by approximately 15-16%.

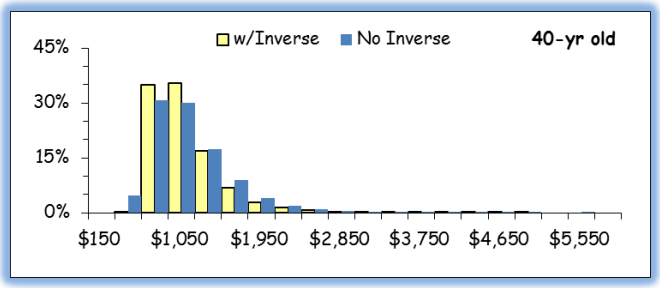



**Panel A: Relative frequency of terminal wealth**

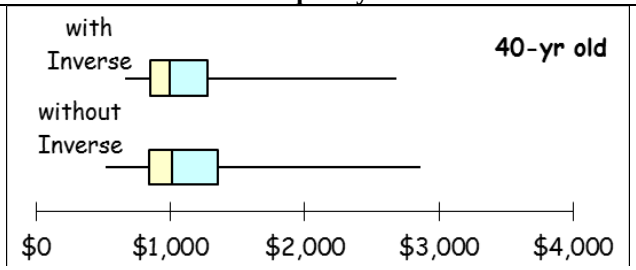



**Panel B: Box plot of terminal wealth**

**Fig. 8.** Terminal wealth distributions for high risk tolerant investor with and without an inverse stock allocation using 1999-2008 return distributions. Distribution assumes starting with $1000, holding investment for 10 years, and negligible transactions costs.

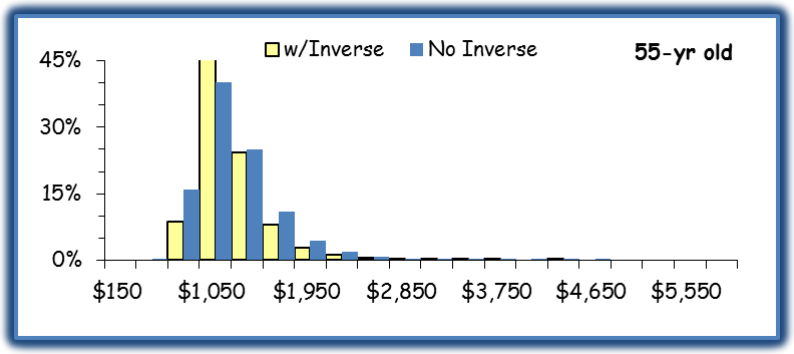



**Panel A: Relative frequency of terminal wealth**




**Panel B: Box plot of terminal wealth**

**Fig. 9.** Terminal wealth distributions for moderate risk tolerant investor with and without an inverse stock allocation using 1999-2008 return distributions. Distribution assumes starting with $1000, holding investment for 10 years, and negligible transactions costs.




**Panel A: Relative frequency of terminal wealth**


**Panel B: Box plot of terminal wealth**

**Fig. 10.** Terminal wealth distributions for low risk tolerant investor with and without an inverse stock allocation using 1999-2008 return distributions. Distribution assumes starting with $1000, holding investment for 10 years, and negligible transactions costs.

**Table 6**
Reduction in mean and standard deviation of terminal wealth using 1999-2008 return distributions for simulation sampling

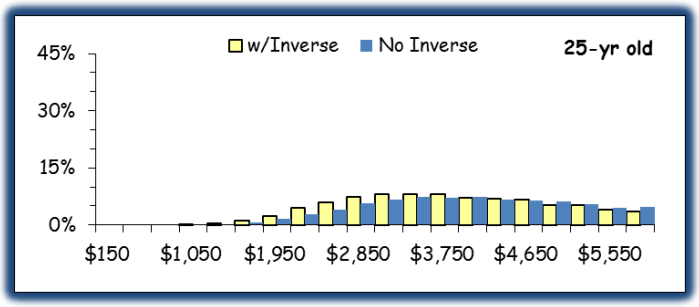
| Terminal Wealth Statistic | 25-yr old | 40 yr-old | 55-yr old |
|---|---|---|---|
| **Standard Deviation** | -16.3% | -17.4% | -18.7% |
| **Mean** | -1.7% | -2.4% | -3.0% |
| **Coefficient of Variation** | -14.9% | -15.3% | -16.2% |

This table shows that the reduction in mean and standard deviation in terminal wealth for each of the three investors identified. Reductions are from 10,000 trials from samples generated from 1999 – 2008.

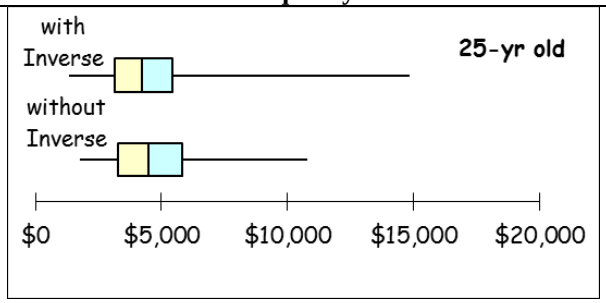
*4.3 Simulated returns using "Optimistic" 1991-2000 return distribution*

The results so far have suggested that the inverse stock fund may be a useful addition to a diversified portfolio of stocks and bonds, since the CV was reduced in both cases. But, the periods previously considered did not exhibit a long term upward trend in stock prices, such as what occurred in the period from 1991 – 2000. Figs. 11-13 simulate a decade of returns under this more bullish time for stocks. For this case, it is not clear if there is any benefit in the use of the inverse fund, as the distribution of total wealth appears to simply have shifted to a lower value without a noticeable reduction in the difference between Q1 and Q3. In fact, the range of the distribution has appeared to have increased significantly.

So, more careful examination of the changes in mean and standard deviation is warranted, which appear in Table 7. Once again, the mean returns are reduced, but the variability measured by the standard deviation is no longer reduced as significantly. Somewhat surprisingly, the coefficient of variation remains negative, suggesting that even during a period of strong positive stock returns, the inverse funds do not provide a detriment to diversification. Another interesting observation from Panel B of Figs. 11-13 is that the use of the inverse fund actually increases the maximum terminal wealth obtained from the simulation significantly, although at a very small probability.

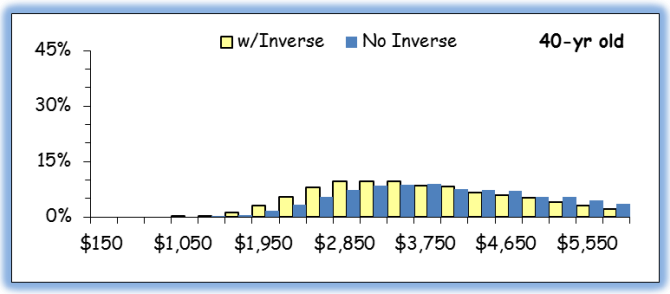



**Panel A: Relative frequency of terminal wealth**

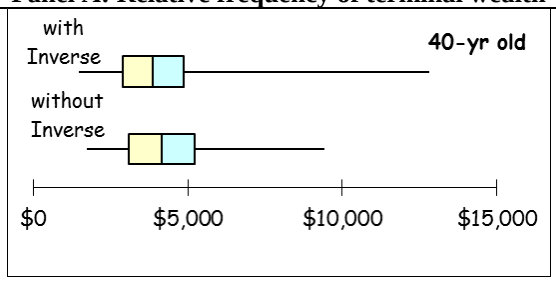



**Panel B: Box plot of terminal wealth**

**Fig. 11.** Terminal wealth distributions for high risk tolerant investor with and without an inverse stock allocation using 1991-2000 return distributions. Distribution assumes starting with $1000, holding investment for 10 years, and negligible transactions costs.

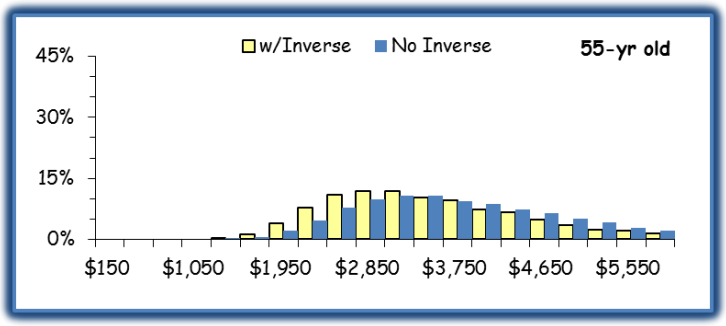



**Panel A: Relative frequency of terminal wealth**




**Panel B: Box plot of terminal wealth**

**Fig. 12.** Terminal wealth distributions for moderate risk tolerant investor with and without an inverse stock allocation using 1991-2000 return distributions. Distribution assumes starting with $1000, holding investment for 10 years, and negligible transactions costs.




**Panel A: Relative frequency of terminal wealth**


**Panel B: Box plot of terminal wealth**

**Fig. 13.** Terminal wealth distributions for low risk tolerant investor with and without an inverse stock allocation using 1991-2000 return distributions. Distribution assumes starting with $1000, holding investment for 10 years, and negligible transactions costs.

**Table 7**

Reduction in mean and standard deviation of terminal wealth using 1991-2000 return distributions for simulation sampling

| Terminal Wealth Statistic | 25-yr old | 40 yr-old | 55-yr old |
|---|---|---|---|
| **Standard Deviation** | -10.3% | -10.4% | -10.5% |
| **Mean** | -9.6% | -9.6% | -9.5% |
| **Coefficient of Variation** | -0.8% | -0.9% | -1.0% |

This table shows that the reduction in mean and standard deviation in terminal wealth for each of the three investors identified. Reductions are from 10,000 trials from samples generated from 1991 – 2000.

## Conclusions and Future Work

We investigated the risks and possible opportunities of a "120 – age" buy-and-hold allocation strategy of stocks and bonds, but also included a small allocation of inverse stock funds. Our assessment was based on three different risk aversion levels, and simulated inverse fund returns assuming a "perfect" inverse performance. Distribution of terminal wealth was obtained by Monte Carlo simulation that selected from observed adjusted price history using three different historical periods, representing three different cases of long-run equity returns. ("Most Recent", "Pessimistic", "Optimistic")

Our finding showed that under all return histories, a buy-and-hold strategy that includes a 10% allocation towards an inverse stock fund provides a diversification benefit by reducing the CV of terminal wealth. The diversification benefit is strengthened when stock total returns are smaller, but does not show an increase in CV when larger positive stock returns are simulated. These results and the potential diversification benefits call into question the current recommendation that inverse stock funds are detrimental to long-term investors, and only beneficial for short term day trading. These findings also suggest that additional analytical and empirical studies are warranted, to better assess the risks and opportunities of inverse stock funds for long-term investors.

Future work in this area can cover several areas. First, a more comprehensive set of cases should be considered. For example, evaluating the effects of 2x/3x levered and -2x/-3x inverse ETFs is suggested to see if the observations found using -1x inverse ETFs considered here persist. Additionally, other return periods of stocks and bonds can be considered and simulated using the methodology proposed here. Further, alternative simulation approaches could be used, such as simulating the price path of stocks and bonds as a stochastic process. Other measures of diversification beyond the CV of terminal wealth could also be examined, such as Sharpe ratio and the diversification "effect" as proposed and applied by Hight (2010). Lastly, tests of statistical significance should be pursued to show the strength of the findings, and verify the results are not simply being observed out of randomness within the simulation.

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