Withdrawal Rate Strategies for Retirement Portfolios: Preventive Reductions and Risk Management[1]

Dr. John B. Mitchell, Professor[2]
Department of Finance and Law
Central Michigan University

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Abstract

This paper builds on the work of Stout and Mitchell (2006), Stout (2008), and Blanchett and Frank (2009) by creating a preventive approach to withdrawal management. Proactive strategies, which reduce the withdrawal rate before there are insufficient funds, are shown to significantly reduce the probability of ruin (shortfall) while maintaining the average withdrawal rate. The paper also explores the micro effects of strategy changes by dividing the simulation iterations into groups which have been positively or negatively affected by any particular change, and demonstrates that conventional reporting of the effectiveness of withdrawal rate management techniques can be improved by examining additional moments of the distribution. Data covers 1926-2008 and the mortality table is extended to 108 years.

1. Introduction

We use proactive management strategies throughout our lives. We change the oil in our car, work out at the gym, and get an annual medical check-up. We do these things because we recognize that anticipating adverse events, and taking corrective action or at least diagnosing them early, increases our chances of avoiding larger more catastrophic problems. The same logic can be applied to managing withdrawals from a retirement portfolio. Retirees who reduce their withdrawals before they no longer have sufficient funds to maintain the current rate are less likely to run out of money. The pain of reduced income can be smoothed over a period of time and the traumatic and life-changing impact of ruin may be avoided or at least deferred.

This paper builds on the withdrawal rate management work of Stout and Mitchell (2006), Stout (2008), and Blanchett and Frank (2009). Stout and Mitchell (2006) employ a set of control limits to manage the withdrawal process and Monte Carlo simulation of security returns and mortality. Stout (2008) optimizes the process. Blanchett and Frank (2009), employing a bootstrapping

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[2] The author thanks G. Thomas Mitchell, FSA for developing the extrapolated life table. The author can be contacted at mitch1jb@cmich.edu or at 328 Sloan Hall, Dept. of Finance and Law, Central Michigan University, Mt. Pleasant, MI 48859. (989) 774 3651

approach, recommends that retirees act proactively to manage withdrawals based on an implied probability of ruin.

This paper adds to the literature by creating withdrawal rate adjustments specific to the individual, extending the mortality table to age 108 to include the experience of additional superannuated retirees, updating security returns through 2008 to include recent market events, and by examining the withdrawal rate and probability of ruin distributions in more detail.

2. Literature Review

Stout and Mitchell demonstrate that withdrawal rate management techniques can provide substantial improvements in the withdrawal rate while simultaneously reducing the probability of ruin (running out of money before a fixed point in time or death). Stout and Mitchell find that a constant real withdrawal rate of 4.5% for 30 years results in a 13.44% chance of ruin based on a 65/35 large-cap stock and intermediate term bond portfolio and data from 1926-2004. While introducing death reduces the probability of ruin to 7.16% (for a 60-year old retiree), management of the withdrawal rate process via several parameters is able to further reduce that risk to 4.43% while increasing the average withdrawal rate to 6.63% and leaving an average legacy of 1.07 times the initial portfolio. The 4.43% risk of a managed withdrawal rate is comparable to the 4.18% probability of ruin found for the same 60-year old if they adopt a fixed 4% real withdrawal rate; however, the managed withdrawal rate allows more than a 50% increase in average income.

Stout and Mitchell (2006) use three types of control limits: 1) portfolio deviation thresholds, 2) withdrawal adjustment rates, and 3) absolute withdrawal rate limits. Portfolio deviation thresholds set portfolio values at which increases or decreases in the real withdrawal rate are made. Withdrawal adjustment rates set the fraction of an allowable increase or decrease which is actually taken. Absolute withdrawal rate limits define the outer limits of the real withdrawal rate.

The primary problem with controls is that they introduce uncertainty about annual income in the form of variability in the real withdrawal rate. Controls also make it more difficult for retirees to understand the decumulation process. Yet such rules are essential to reducing failure and preventing excess accumulations, as evidenced by Stout and Mitchell (2006). The decrease in the probability of ruin stems from the ability to reduce the withdrawal rate when underperforming and the increase in the average withdrawal rate is derived from the opportunity to consume what were formerly excess accumulations. Different controls are needed for these two different purposes.

Stout (2008) demonstrates, through a more rigorous and systematic examination of the optimization process for the same set of management rules employed by Stout and Mitchell (2006), that the risk can be reduced to 3.75% for a 60-year old retiree. Stout determines an optimal portfolio allocation of a two-class (large cap stocks and intermediate-term bonds) portfolio (approximately 60% equity) and parameters, which include a floor of 3% and initial withdrawal rate of 4.5%, for managing the decumulation process. Stout, using 1926-2006 data, finds that the same retiree, without management of withdrawal rates, has a 7.50% probability of ruin, only slightly higher than the 7.16% rate calculated in the earlier Stout and Mitchell (2006).
Mulvey and Purcell (2008) provide a review of much of the literature on the topic. Numerous authors study the question of sustainable withdrawal rates and withdrawal rate management.

Bengen (2001) employs floor (-10%) and ceiling (+25%) limits. Pye (2000) withdraws the lower of the previous amount (in real terms) or the amortized current portfolio value using the plan length and expected portfolio rate of return. Guyton (2004) imposes systematic decision rules (restrictions) on withdrawal rate increases and subsequent make-ups for foregone increases. Guyton and Klinger (2006) expand upon Guyton's earlier article by modifying the withdrawal freezes, eliminating his inflation rule, and adding capital preservation and prosperity rules. The result is a significant improvement in performance at the expense of much more complexity. Klinger (2007) follows this work with a model based on a modified withdrawal rule and capital preservation and prosperity rules, both of which involve triggering limits and rates.

Many studies employ fixed real withdrawal rates (WR) in the range of 4-5% and report various failure rates over a 30-year planning horizon depending on methodology: Bengen (1997) 0% at a 4.08% WR and 12% at a 4.5% WR; Ameriks, Veres, and Warshawsky (2001) 12.6% at a 4.5% WR; Cooley, Hubbard, and Walz (1998) 24% at a 5% WR but as low as 2% for a 4% WR; Tezel (2004) 3.16% at a 4.5% WR; and Ervin, Filer and Smolira (2005) 6% at a 4% WR, 32% at a 5% WR, 54% at a 6% WR and 73% at a 7% WR. In summary, low risk at a 4% WR but substantial risk at even slightly higher WR based on 30-year planning horizons.

Milevsky (2005) reports that a 60-year old retiree, considering mortality, has a 13.7% probability of risk of failure with a 4% WR and 22.9% failure with a 4.5% WR. Milevsky assumes an artificial, but reasonable, 7% mean return and 20% standard deviation of returns.

Blanchett and Frank (2009) studies 30-year periods with both fixed rate and with annual withdrawal rate adjustment. Blanchett and Frank adjusts the withdrawal rate by a fixed 3% (of the current rate) up or down depending on a calculated probability of ruin and rules tied to remaining time. A 4% initial withdrawal rate yields a 4.07% probability of ruin if the withdrawal rate is fixed and 2.65% if managed. 5% causes a 16.44% probability of ruin if fixed versus 4.57% if managed and a 6% initial withdrawal rate generates a 39.01% probability if fixed and 9.83% if managed. Quoted probabilities of ruin are given for 60% equity portfolios although Blanchett and Frank provides results for four different equity allocations, time spans ranging from 10 to 50 years, and initial withdrawal rates ranging from 4 to 12%. Blanchett and Frank does not provide average withdrawal rates over the 30-year planning period but does provide 95th, 90th, 80th, and 50th percentile rates for the final year. For example, the managed strategies referred to above yield median withdrawal rates in the 30th year of 8.83%, 8.58%, and 8.21% for initial withdrawal rates of 4%, 5%, and 6% respectively.

Blanchett and Frank doesn't explicitly incorporate mortality which may overstate the probability of ruin, for example, for 50-year olds with less than a 30-year expected remaining lifespan. They also use a one-size-fits-all 3% (of the previous withdrawal rate) annual adjustment and use bootstrapping based on monthly data. Bootstrapping limits the distribution of security returns to only those events experienced during the data collection period.

However, Blanchett and Frank (2009) adds significantly to the literature by exploring a proactive strategy based on annually managing the probability of ruin. Just as Blanchett and Frank build upon Stout and Mitchell (2006), this paper builds upon Blanchett and Frank (2009).

Spitzer, Strieter, and Singh (2008) likewise uses bootstrapping of annual data (1926-2003). They test for sustainability with 13 different equity allocations, fixed withdrawal rates ranging from 3% to 5.5%, and four adaptive withdrawal strategies over a 30-year withdrawal period. No strategy resulted in zero probability of ruin. Fixed withdrawal rates yielded 1% to 25% probabilities of ruin at optimal equity allocations ranging from 40% to 80% at various withdrawal rates. All four of their adaptive strategies, and their fixed 4% withdrawal rate yielded 5 - 8% probabilities of ruin at various optimal equity allocations of 45% to 55% and average withdrawal rates of 4 – 5%. Their best strategy appears to be one they label V1 with a 5.73% average withdrawal rate and 6.3% probability of ruin at a 55% equity allocation. Spitzer et. al., use simple adaptive strategies, which, like Blanchett and Frank (2009), involve fixed withdrawal rate adjustment amounts based on portfolio performance. Also like Blanchett and Frank, they do not directly incorporate mortality.

All of the existing studies of withdrawal rate management indicate benefits in terms of an improved risk-return relationship and therefore encourage the search for improved methods. This paper demonstrates one such improvement, a simulation-based proactive withdrawal rate adjustment model incorporating mortality and tailored to the individual situation.

3. Motivation for Research

This paper contributes to the development of improved withdrawal rate strategies in several ways.

1) The model developed in this research anticipates the difficulty a retiree faces in maintaining a particular withdrawal rate and tailors the amount of adjustment to the individual situation. The adjustment rule requires that retirees maintain some (various) multiple of the amortized amount of their expected withdrawals based on their expected remaining lifespan and historic security returns.

2) The research considers mortality in calculating the probability of ruin and extrapolates the CDC mortality table to age 108 so as to protect the most vulnerable retirees.

3) The research uses annual data from 1926-2008 to incorporate the dramatic effects of recent market instability.

4) Micro effects are studied to determine how many retirees are helped, or hurt, by a particular withdrawal rate technique. Micro effects are studied both by capturing and studying individual iterations of the simulation and by examining additional moments of the distribution.

Most studies of withdrawal rate management employ a fixed, often 30-year, planning horizon. Such studies overstate the probability of ruin for older retirees with shorter remaining life spans. Similarly, these same studies may significantly understate the probability of ruin for younger retirees or due to changes in lifespan allowed by advances in medical technology. One of the earliest known mortality tables shows that in 1662 London, 60% of people died before age 16 and virtually no one lived beyond age 76 [See Mlodinow (2008), page 152]. Over time, significant improvements in expected lifespan have caused the average child born in the U.S. to have an expected lifespan of 77.8 years according to the 2004 CDC Life Table. This research simulates the game of life by randomly sampling from the mortality table in the same manner as Stout and Mitchell (2006). However, the mortality table has been extrapolated to age 108 to more conservatively approximate the percent of superannuated retirees who suffer financial ruin.

Many studies, recently Blanchett and Frank (2009) and Spitzer, Strieter, and Singh (2008), employ a bootstrapping technique in which returns are randomly drawn from a historic distribution of either monthly or annual returns from some sample period. The weakness of this technique is that it effectively truncates the distribution based on historic experience. If 2008 taught us nothing else, it drove home the point that the future can change very quickly and may bear little resemblance to the past. This research employs a Monte Carlo simulation technique, as used in Stout and Mitchell (2006), based in this case on annual returns experienced from 1926-2008. In such a model, more extreme events than those historically experienced may occur. However, even Monte Carlo simulation techniques have their limitations. Heteroskedasticity may render the distributions built into the model obsolete. Further research might focus on how robust the model is in higher variance time periods. The use of Monte Carlo simulation will facilitate such a line of research.

The fourth modification built into this research is a study of the effects of withdrawal rate management technique changes on various sub-groups of the population. A change in withdrawal management strategy may help some retirees both in terms of reduced incidence of ruin and higher withdrawal rates. However, this may come at the expense of other retirees who never experienced ruin but who are penalized with a lower withdrawal rate under the new management strategy. Micro effects also include studying various percentiles of the distribution to identify the fraction of retirees who may not ruin but do suffer potentially disastrous low withdrawal rates.

The four modifications of the Stout and Mitchell (2006) and Stout (2008) research: higher downward thresholds, extrapolated mortality table, updated data, and studying micro-effects; provide a more conservative and detailed view of the risk-return relationship and an effective technique for improving that relationship. The probability of ruin can be significantly reduced from the levels shown in previous research while maintaining, or enhancing, the withdrawal rate.

This paper builds upon Blanchett and Frank (2009) by tailoring the adjustments in a proactive withdrawal rate adjustment model to the individual circumstances including specific consideration of expected life and exact portfolio value.

Existing research does not address the question of acceptable probabilities of failure (running out of money before the end of the planning horizon) although the need to do so is noted both by Terry (2003) and Bengen (2006). In the absence of a scientifically derived acceptable probability of ruin, this research helps reduce the level of risk experienced by retirees while maintaining withdrawal rates for the majority of retirees that are substantially higher than the traditional 4%.

4. The Withdrawal Rate Management Model

The techniques for managing withdrawals are as follows:

- Upward and Downward Thresholds
- Maximum and Minimum Withdrawal Rate
- Upward and Downward Adjustment Rates

The Upward (Upthrsh) and Downward (Dnthrsh) Thresholds are multiples of the present value (at historic rates of return for the retiree's expected planning horizon (remaining expected lifespan). An Upward Threshold of 2.6, for example, would require the retiree to have 2.6 times the amount needed to fund expected withdrawals before they increase their withdrawal rate. The amount needed at time (t) is calculated as the present value as an annuity due of the current withdrawal rate discounted at an expected rate of return (r) equal to historic rates for all time periods equal to their expected remaining lifespan (L).

If $PORT_t > (Upthrsh \times PVIFA_{DUE(L,r)} \times PORT_0 \times WR_{t-1})$ (1)

then $WR_t = PORT_t / (Upthrsh \times PVIFA_{DUE(L,r)} \times PORT_0)$. (2)

Otherwise, $WR_t = WR_{t-1.}$ (3)
Similarly, If $PORT_t < (Dnthrsh \times PVIFA_{DUE(L,r)} \times PORT_0 \times WR_{t-1})$ (4)

then $WR_t = PORT_t / (Dnthrsh \times PVIFA_{DUE(L,r)} \times PORT_0)$. (5)

Otherwise, $WR_t = WR_{t-1.}$ (6)

This research differs from Stout and Mitchell (2006) and Stout (2008) by allowing the Downward Threshold to exceed a value of 1. A Downward Threshold greater than 1 would require the retiree to reduce their withdrawal rate even though they have an amount in their portfolio equal to the present value of the current withdrawal rate discounted as an annuity for their expected remaining lifetime at the historic rate of return for such time periods. A Downward Threshold of 1.5, for example, would require a 50% excess over that discounted value. It is this higher Downward Threshold which constitutes the proactive aspect of the withdrawal management strategy. Retirees are required to anticipate the need to reduce their withdrawals. Downward Thresholds above 1 are mentioned (their definition refers to a value of zero but it means the same as the value of 1 referred to here), but not pursued, in Stout and Mitchell (2006). The higher Downward Threshold is similar, in this respect, to the technique employed by Blanchett and Frank (2009) in which the implied probability of ruin controls when increases and decreases in the withdrawal rate occur. However, the mechanism in this research requires an adjustment amount individually tailored to retiree's portfolio and their expected remaining life. Blanchett and Frank in effect reduce remaining life (planning horizon) by one year for each year of age. However, expected remaining life decreases less rapidly. See Table 1. For example, a 60-year old has an expected remaining life of 22.51 years and 10 years later they still have an expected remaining life of 15.15 years, only 7.36 years less.

The Maximum (WR_{MAX}) and Minimum (WR_{MIN}) Withdrawal Rates are the highest and lowest withdrawal rates (WR_t) allowed for an individual retiree. The Minimum Withdrawal Rate may be violated only in the year of ruin, as the retiree takes remaining funds from the portfolio.

If $WR_t > WR_{MAX}$, then $WR_t = WR_{MAX.}$ (7)

If $WR_t < WR_{MIN}$, then $WR_t = WR_{MIN.}$ (8)

Upward (Uppart) and Downward (Dnpart) Adjustment Rates are the fraction of available increase or decrease taken by the retiree during a particular period. For example, if a retiree amortizes their portfolio over their expected remaining lifespan at historic rates of return and finds the portfolio could sustain a 10% withdrawal rate (WR_{PRELIM}) as compared to a current 6% withdrawal rate, a 40% Upward Adjustment Rate would allow them to only increase the withdrawal rate to 7.6% i.e. (6% + (.4 x (10% - 6%)).

If $WR_{PRELIM} > (WR_{t-1} + (Uppart \times (WR_{PRELIM} - WR_{t-1}))),$ (9)

then $WR_t = (WR_{t-1} + (Uppart \times (WR_{PRELIM} - WR_{t-1}))).$ (10)

Similarly, if $WR_{PRELIM} < (WR_{t-1} - (Dnpart \times (WR_{t-1} - WR_{PRELIM}))),$ (11)

then $WR_t = (WR_{t-1} - (Dnpart \times (WR_{t-1} - WR_{PRELIM}))).$ (12)

5. Data and Simulation Procedure

Simulation of the withdrawal phase of retirement planning is performed in GoldSim utilizing the same procedure found in Stout and Mitchell (2006) and Stout (2008). All simulations consist of 50,000 iterations with the same seed. The return data for the study is from SBBI for the period 1926-2008. Portfolios are limited to a single combination of the Large-cap Stock (65%) and Intermediate-term Government Bond (35%) indexes. This particular portfolio was chosen to mimic the recommendation for 60-year olds in Stout (2008). Average (arithmetic) real return and (standard deviation) are: Large-cap Stocks 8.47% (20.57%) and Intermediate-term Government Bonds 2.57% (6.86%). Natural logs are employed in the simulation to normalize the distribution. All results are reported for 60-year old retirees except as noted. Ruin is defined as the year in which a retiree (iteration) falls below the minimum allowable withdrawal rate. The number of years of ruin is the time in ruin until death. The average withdrawal rate is defined as the average over all years to death, including years of zero withdrawals due to ruin. The portfolio is assumed to be rebalanced each year at zero cost and all withdrawals are calculated on a before-tax basis.

Table 1 contains the mean (expected) remaining lifespan at various ages and standard deviation of the remaining lifespan based on the 2004 Life Table of the Centers for Disease Control and Prevention. The CDC Life Table annual death rates have been extrapolated linearly from age 98 - 99 rates and carried forward until the age (108) at which truncation of the table reproduces the expected lifespan on the CDC Life Table for age 100. This increases the standard deviation for ages approaching 100. Stout and Mitchell (2006) and Stout (2008) effectively kill off all retirees at age 100 by adopting a Life Table truncated at age 100. This difference, as adopted in this paper, will significantly affect, and more accurately reflect, risk for the oldest and most vulnerable retirees. Table 2 shows the extrapolated portion of the Life Table as employed in the simulations.

Historic average returns for various holding periods (1926-2008 data) are also reported in Table 1 along with the standard deviation of those returns. The standard deviation of portfolio returns increases with age due to less time diversification of returns. A year of less than average (or even negative) returns is less likely to be reversed by later, above average, returns. While holding period returns behave in a somewhat random manner (a function of the sample period); shorter holding periods do have higher standard deviations.

Age	Expected Remaining Lifespan	Std. Dev. Of Remaining Lifespan	Historic Returns	Standard Deviation of Historic Returns
50	30.87	11.96	5.42%	.97%
55	26.61	10.98	5.57%	1.45%
60	22.51	10.12	5.64%	2.10%
65	18.69	9.15	5.68%	2.67%
70	15.15	8.10	5.70%	3.19%
75	11.95	6.99	5.77%	3.62%
80	9.15	5.88	5.76%	4.11%
85	6.83	4.78	5.68%	4.60%
90	4.99	3.77	5.65%	5.52%
95	3.60	2.88	5.72%	6.42%
100	2.60	2.09	5.90%	7.68%

Table 1
Moments of the distributions of remaining lifespan and historic returns (1926-2008)

Age	Expected Remaining Lifespan	Standard Deviation Remaining Life
100	2.6	2.09
101	2.4	1.92
102	2.2	1.74
103	2.1	1.54
104	1.9	1.30
105	1.6	1.02
106	1.3	0.67
107	0.8	0.26
108	0.0	0.00

Table 2
Extrapolated life table in years

6. Results

Comparison to Stout and Mitchell (2006)

Stout and Mitchell (2006) does not establish an optimal solution. They report, for example, for a 4.5% Initial Withdrawal Rate, 2.4 Upward Threshold and 1.0 Downward Threshold (both equivalent to the 1.4 and 0.0 reported in Stout and Mitchell), .2 Upward Adjustment Rate and 1.0 Downward Adjustment Rate, 40% Maximum Withdrawal Rate and 3% Minimum Withdrawal Rate. The above combination of controls yields a 6.63% average withdrawal rate, 4.33% probability of ruin to age 100, and average ending portfolio 1.07 times the beginning amount based on 1926-2004 data and their seed for 10,000 iterations. The same controls, replicated for 1926-2008 data, the same seed used for all other simulations in this paper, and 50,000 iterations yields a 6.47% average withdrawal rate, 8.68% probability of ruin to age 100 (9.38% to age 108), and average ending portfolio .88 times the beginning amount. Clearly, recent returns, particularly 2008, require an increased perception of risk and make comparison to earlier studies difficult. The greatly increased probability of ruin also calls into question the 65% equity portfolio employed by Stout and Mitchell (2006), recommended by Stout (2008) when a retiree chooses a 5% Initial withdrawal Rate, and which is so common in the literature. The higher risk environment prevailing after the 2008 market instability may lead to lower optimal equity allocations.

Comparison to Stout (2008)

Stout (2008) reports that a 60-year old retiree with a 5% Initial Withdrawal Rate, no Maximum Withdrawal Rate, 3% Minimum Withdrawal Rate, 2.734 Upward Threshold, 1.0 Downward Threshold, .312 Upward Adjustment Rate, and 1.0 Downward Adjustment Rate would experience an average withdrawal rate of 6.25%, average ending portfolio of 1.19, and a 5.88% probability of ruin. Applying the same parameters (with the exception of limiting the Maximum Withdrawal Rate to 40%) to 1926-2008 data yields a 6.47% average withdrawal rate, ending portfolio of .86, and 10.31% probability of ruin to age 108. Truncating the process at age 100 reduces the probability of ruin to 9.65%. It is interesting to note that the controls proposed in Stout and Mitchell (2006) give better results, based on data through 2008, than the optimal solution proposed by Stout (2008). The average withdrawal rate is the same (6.47%) while the probability of ruin to age 100 is lower by . 97% (8.68% vs. 9.65%) based on the controls in Stout and Mitchell (2006). What is optimal today may not be tomorrow; presumably due to heteroskedasticity.

Table 3 provides results for Downward Thresholds varying from 1 (Stout) to 2. Downward Thresholds of 1.5 and higher effectively limit the Initial Withdrawal Rate because the higher threshold multiplied by the $PVIFA_{DUE}$ for the expected life (initially 23 years) and the 5% intended Initial Withdrawal Rate exceeds the Initial Portfolio. Therefore, Table 3 shows the actual (maximum) Initial Withdrawal Rate for each Downward Threshold.

Table 3 demonstrates that a Downward Threshold greater than 1 is effective at preventing ruin. This paper is not intended to optimize the decision process. Indeed, in the absence of an acceptable probability of ruin, the choice is subjective. However, a decrease of .32% (6.47% to 6.15%), a 4.9% rate of decrease, in the average withdrawal rate is offset by more than an 8% (10.31% to 2.17%) decrease in the probability of ruin, a 79% rate of decrease. The slightly lower average withdrawal rate experienced at higher Downward Thresholds is also reflected in the slightly higher average ending portfolio as most retirees leave a larger reserve at the time of their death. The higher Downward Threshold is not successful, however, in significantly extending the time until first ruin or the longest time in ruin. Time to first ruin and longest time in ruin both represent a worst case scenario rather than averages. The average years in ruin does decrease from .49 years to .09 years (not shown in table) as the Downward Threshold increases from 1 to 2. However, it should also be remembered that those years of ruin are actually experienced, concentrated on, a relatively few who may endure close to 30 years of ruin at the lower Downward Thresholds. The sub-set of retirees who do experience ruin see a decrease from 4.71 years to 4.20 years. Far fewer experience ruin at the higher Downward Threshold and those who do ruin suffer for a shorter period of time.

Downward Threshold	Initial Withdrawal Rate	Average Withdrawal Rate Mean (Std Dev)	Average Ending Portfolio Mean (Std Dev)	Probability Of Ruin Age 108 (Age 100)	Year of First Ruin (Max Years of Ruin)
1.0	5.00%	6.47% (3.04%)	.86 (1.05)	10.31 (9.65)	13 (29)
1.1	5.00%	6.45% (3.05%)	.87 (1.04)	8.54 (7.88)	13 (28)
1.2	5.00%	6.42% (3.05%)	.89 (1.04)	7.02 (6.28)	13 (28)
1.3	5.00%	6.40% (3.06%)	.91 (1.04)	5.80 (5.05)	13 (27)
1.4	5.00%	6.37% (3.08%)	.94 (1.04)	4.79 (4.12)	13 (27)
1.5	4.82%	6.34% (3.12%)	.97 (1.05)	4.01 (3.43)	13 (26)
1.6	4.52%	6.30% (3.17%)	1.01 (1.06)	3.40 (2.90)	14 (25)
1.7	4.25%	6.26% (3.21%)	1.04 (1.07)	2.92 (2.54)	14 (24)
1.8	4.02%	6.22% (3.24%)	1.08 (1.08)	2.59 (2.25)	14 (24)
1.9	3.81%	6.19% (3.27%)	1.11 (1.11)	2.33 (2.04)	14 (24)
2.0	3.62%	6.15% (3.28%)	1.13 (1.13)	2.17 (1.93)	14 (24)

Table 3
Effect of Downward Threshold on Initial Withdrawal Rate, average withdrawal rate, average ending portfolio, and probability of ruin
40% Maximum Withdrawal Rate, 3% Minimum Withdrawal Rate, 2.734 Upward Threshold, .312 Upward Adjustment Rate, and 1.0 Downward Adjustment Rate for retiree age 60

Figure 1 incorporates the data from Table 3 for retirees at age 60 (bottom line) and similar results for age 65 (top line). Downward Thresholds of 1 are on the right end of the curve. As the Downward Threshold rises, both the average withdrawal rate and the probability of ruin fall. Data for age 65 was compiled using the same controls as for age 60 rather than the optimal controls in Stout (2008) so as to preserve comparability between ages. The fact that the age 65 curve is higher and to the left of the age 60 curve indicates the lower risk per unit of return made possible by the shorter expected withdrawal period. Individuals who delay retirement can achieve higher withdrawal rates with less risk as shown in other studies. What is of interest here is that the curves are very close to parallel indicating a similar risk-return effect from any modification of the Downward Threshold.

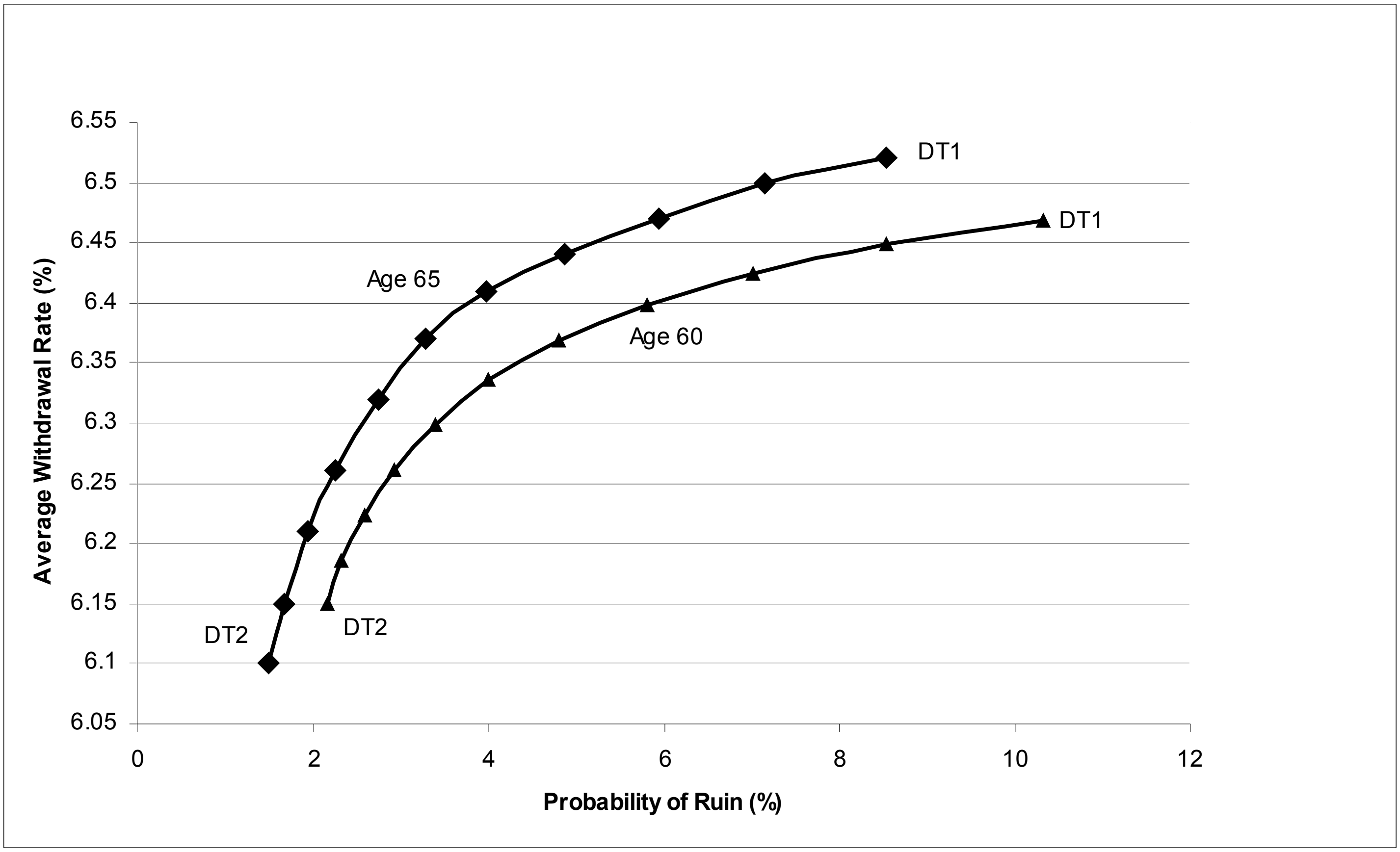

Figure 1
Effect of Downward Threshold on average withdrawal rate and probability of ruin for ages 60 and 65
40% Maximum Withdrawal Rate, 3% Minimum Withdrawal Rate, 2.734 Upward Threshold, .312 Upward Adjustment Rate, and 1.0 Downward Adjustment Rate

Table 4 provides results for various points in the distribution of the average withdrawal rate. Note first that the median is approximately 1% (on average 1.09%) below the mean provided in Table 3 for each Downward Threshold. The distribution of retiree's average withdrawal rate is skewed due to some retirees approaching the 40% Maximum Withdrawal Rate. In fact, some retirees reach the annual 40% maximum in as little as 12 years. The median is probably a more appropriate value to report to retirees since it better reflects what they should expect as individuals. This indicates a problem with how results are commonly reported in retirement planning studies. The amount of skewness is related to the value for the Maximum Withdrawal Rate. 40% is, of course, equivalent to $400,000 in real terms on a $1,000,000 Initial Portfolio. While a 40% Maximum Withdrawal Rate is instrumental in reducing the average ending portfolio and giving retirees an idea of just how good their retirement might get, it may not reflect the spending pattern of many retirees even if they can afford it. Note in Table 4 that the median increases as the Down Threshold increases (up to 1.6). Therefore, the typical retiree will actually experience an increase in their withdrawal rate at higher Down Thresholds, in contrast to the evidence provided by the mean average withdrawal rate.

Reducing the Maximum Withdrawal Rate (as a way to reduce skewness and kurtosis) does not significantly affect the probability of ruin as shown in an unpublished paper by the author and available on SSRN at http://ssrn.com/abstract=1371695. This lack of effect on the probability of ruin is due to the fact that the population likely to experience ruin, those with underperforming portfolios, is not likely to be affected by the Maximum Withdrawal Rate.

Downward Threshold		Lowest x% of distribution					
	Median	5%	4%	3%	2%	1%	.1%
1.0	5.15	3.63	3.48	3.28	2.95	2.63	1.92
1.1	5.19	3.61	3.49	3.32	3.05	2.66	1.92
1.2	5.23	3.57	3.47	3.33	3.10	2.68	1.92
1.3	5.27	3.52	3.43	3.32	3.14	2.69	1.92
1.4	5.29	3.47	3.39	3.29	3.16	2.71	1.93
1.5	5.31	3.41	3.33	3.25	3.15	2.72	1.93
1.6	5.31	3.34	3.28	3.21	3.12	2.73	1.93
1.7	5.27	3.28	3.32	3.16	3.09	2.74	1.92
1.8	5.23	3.22	3.17	3.12	3.07	2.75	1.91
1.9	5.19	3.17	3.12	3.08	3.05	2.75	1.90
2.0	5.16	3.12	3.09	3.06	3.03	2.76	1.89

Table 4
Effect of Downward Threshold on average withdrawal rate of lowest (x) percent of retirees
40% Maximum Withdrawal Rate, 3% Minimum Withdrawal Rate, 2.734 Upward Threshold, .312 Upward Adjustment Rate, and 1.0 Downward Adjustment Rate

Table 4 also provides the average withdrawal rate that distinguishes the lowest (x%) of the distribution. For example, at a Downward Threshold of 1.0 the dividing line for the lowest 5% of retirees in terms of average withdrawal rate is 3.63%, while an average withdrawal rate of 3.48% separates the lowest 4%. Note that the Downward Threshold has very little impact on average withdrawal rate for the lowest .1% of retirees and modest impact until reaching the 4th to 5th percentiles of retirees. Note also that a Downward Threshold of 1.5 is tied with 1.6 for the highest median but has slightly higher average withdrawal rates for the lower percentiles. The real benefit of the higher Downward Thresholds is that they save some retirees from ruin even though the higher thresholds do not significantly affect the withdrawal rate on average. At a Downward Threshold of 1.0 the 10.31% (Table 3) probability of ruin means that all (or virtually all) retirees with a 3.63% average withdrawal rate (and most likely somewhat higher rates) do ruin at some point. At the other extreme, a Downward Threshold of 2.0 has a probability of ruin of only 2.17% (Table 3) and therefore retirees with even a 3.06% average withdrawal rate probably avoided ruin. It is difficult to change the average withdrawal rate except by keeping retirees ‘in the game’. The goal must be to help retirees have reasonable expectations and a mechanism for avoiding overspending their means.

Table 5 demonstrates that while the median average withdrawal rate from Table 4 is more than 5%, close to 50% of all retirees fail to achieve a 5% average withdrawal rate. The percentage failing to average 5% is lowest (but not significantly so) at a downward Threshold of 1.6. There is a greater, and most pronounced, impact, on the percent failing to achieve 4%, i.e. lower Downward

Thresholds are better. Higher Downward Thresholds trap some retirees at withdrawal rates between 3 and 4% when using a 3% Minimum Withdrawal Rate. Although this reduction in consumable income appears unfavorable, it is the mechanism that prevents those same retirees from ruin. For example, at a Downward Threshold of 2.0 only 1.37% of retirees fail to average a 3% withdrawal rate. This comes close to accounting for all retirees experiencing ruin. In contrast, at a Downward Threshold of 1.0 even some retirees averaging over 4% experienced ruin as evidenced by the probability of ruin (10.31%) exceeding the percentage with less than a 4% average withdrawal rate (8.69%). They feasted (relative to their capacity) for some years but also faced at least some years of ruin.

Downward Threshold	Withdrawal Rate			
	5%	4%	3%	2%
1.0	49.30	8.69	2.02	.14
1.1	48.73	9.46	1.88	.14
1.2	47.93	10.48	1.75	.14
1.3	47.10	11.80	1.66	.14
1.4	46.36	13.41	1.59	.14
1.5	45.16	15.35	1.52	.14
1.6	44.87	17.64	1.47	.14
1.7	45.07	20.17	1.42	.14
1.8	45.86	23.03	1.40	.14
1.9	46.79	26.96	1.37	.14
2.0	47.41	27.38	1.37	.14

Table 5
Effect of Downward Threshold on percent of retirees not reaching various withdrawal rates
40% Maximum Withdrawal Rate, 3% Minimum Withdrawal Rate, 2.734 Upward Threshold, .312 Upward Adjustment Rate, and 1.0 Downward Adjustment Rate

Another perspective on the relationship between Downward Threshold, average withdrawal rate, and the probability of ruin is the implication that at a Downward Threshold of 1.0 it takes more than an average withdrawal rate of 4% to avoid ruin (the probability of ruin was 10.31% which exceeds the 8.69% of the population with 4% or lower average withdrawal rates) while at a Downward Threshold of 2.0 it only takes somewhat more than 3% (the probability of ruin was 2.17%). The average withdrawal rate for those not hitting the maximum is, to a great extent, a function of their cumulative returns, a result which is not affected by the withdrawal strategy (other than portfolio allocation) except to the extent the strategy affects avoiding ruin.

Table 6 provides year-by-year average, median, and several other moments of the average withdrawal rate distribution for downward thresholds of 1.0 and 1.5. The mean withdrawal rate during a particular year is lower for the higher Downward Threshold (1.5 versus 1.0) until age 76 while the median is lower until age 77. The rising mean and median, especially at higher ages, raises questions from a life-cycle perspective about available funds for consumption. Retirees may prefer greater consumption at younger ages when they are more active, and therefore prefer the lower Downward Threshold, with recognition that doing so causes reduced consumption if they superannuate. The higher Downward Threshold, however, is rewarded with increasing gains in both mean and median over time.

Maximum and Minimum Withdrawal Rates are not shown in Table 6. The Minimum Withdrawal Rate of 3% is reached at age 63 (1.0) and age 61 (1.5). Note that the Downward Threshold of 1.5 better prevents almost all retirees from experiencing ruin, as evidenced by higher sub-3% withdrawal rate values in the 5th percentile column.

Age	DT1 Mean	DT1 5%	DT1 25%	DT1 Median	DT1 75%	DT1 95%	DT1.5 Mean	DT1.5 5%	DT1.5 25%	DT1.5 Median	DT1.5 75%	DT1.5 95%
60	5.00	5.00	5.00	5.00	5.00	5.00	4.82	4.82	4.82	4.82	4.82	4.82
65	5.17	3.92	4.53	5.08	5.71	6.75	4.67	3.23	3.76	4.41	5.31	6.99
70	5.93	3.34	4.24	5.40	7.08	10.31	5.63	3.12	3.76	4.88	6.75	10.67
75	6.99	1.91	4.16	6.42	9.23	14.02	6.92	1.73	3.99	6.31	9.23	14.24
80	8.39	1.33	4.10	7.37	11.68	18.96	8.53	1.43	4.27	7.55	11.83	19.01
85	9.78	0.94	4.04	8.31	14.18	23.67	10.16	1.27	4.63	8.89	14.50	23.45
90	11.64	0.69	4.18	9.76	17.55	28.91	12.37	1.25	5.39	10.93	18.10	28.41
95	13.35	0.50	4.23	11.18	20.91	33.12	14.42	1.24	6.15	12.99	21.55	32.32
100	16.12	0.58	5.42	14.42	25.81	36.74	17.63	1.32	7.70	16.75	26.97	36.68
108	20.82	1.84	10.31	21.18	31.54	38.71	23.39	3.00	13.53	24.88	34.13	39.33

Table 6
Annual mean, median, 5th, 25th, 75th, and 95th percentiles of the average withdrawal rate distribution for 1.0 and 1.5 Downward Threshold for selected ages
40% Maximum Withdrawal Rate, 3% Minimum Withdrawal Rate, 2.734 Upward Threshold, .312 Upward Adjustment Rate, and 1.0 Downward Adjustment Rate

The Maximum Withdrawal Rate of 40% is reached at age 72 for both Downward Thresholds. The Downward Threshold is virtually irrelevant, except for possibly affecting the Initial Withdrawal Rate, near the upper end of the distribution. The retirees at the top end of the distribution are, as discussed before, essentially a different population.

Both Downward Thresholds keep the middle 50% of the population near, or above, a 4% withdrawal rate. The Downward Threshold of 1.5, however, forces most retirees to reduce their withdrawal rate early in their retirement as evidenced by the lower mean and median. This is the price for reducing the risk of ruin in old age.

The various moments of the distribution shown in tables 4, 5, and 6 demonstrate the risk-reduction benefits of a higher Downward Threshold; and that those benefits come at the expense of reduced consumption early in retirement. Table 7 provides information about how many retirees are helped, or hurt, by switching from a Downward Threshold of 1.0 to 1.5. Table 7 demonstrates that approximately 32% of retirees experience an increase in the average withdrawal rate. This happens primarily by restraining consumption early in retirement and enjoying the long-term benefits of security returns. Of this 32%, about 26% have no reduction in ruin but benefit from a .40% increase in the withdrawal rate. About 6% of retirees experience both a decrease in ruin (4.58 years on average) and an increase in their withdrawal rate (.32%). Of the 68% of retirees with a decrease of withdrawal rate, about 3% offset their .16% decrease in withdrawal rate with a 2.2 year decrease in ruin. The choice is subjective, but they likely would favor such a plan as having a small price for avoiding over 2 years of ruin. The downside is that over 65% of retirees see a decrease of .39% in their withdrawal rate with no offsetting reduction in ruin. Finally, .02% of retirees (1 of the 5,000 sampled) have both a .11% decrease in withdrawal rate and a 1-year increase in ruin.

Ruin	AWR Increased (% change)	Average Change In Years of Ruin	AWR Decreased (% change)	Average Change in Years of Ruin
Increased .02%	0%		.02% (-.11%)	+1
Unchanged 91.56%	26.54% (+.40%)	N/A	65.08% (-.39%)	N/A
Decreased 8.42%	5.62% (+.32%)	-4.58 years	2.74% (-.16%)	-2.20 years
	32.16%		67.84%	

Table 7
Effects of withdrawal rate strategy on sub-populations.
Changing from Down Threshold 1.0 to 1.5, based on sample of 5,000 retirees
40% Maximum Withdrawal Rate, 3% Minimum Withdrawal Rate, 2.734 Upward Threshold, .312 Upward Adjustment Rate, and 1.0 Downward Adjustment Rate

The coefficient of variation, not reported in Table 7, is found to increase for 64% of retirees as the standard deviation of the average withdrawal rate increases and the average withdrawal rate decreases. 6% of retirees are not affected and 30% see a decrease in their coefficient of variation. The problem with a coefficient of variation approach is that it only addresses return and its relative variability and ignores the greater threat of ruin.

The Curious Case of Retiree 1551

How could an increase in the Downward Threshold both reduce the average withdrawal rate and shorten the time to ruin for that one unlucky retiree? It is possible for the higher Downward Threshold to require a reduction in the withdrawal rate and thereby cause an increase in the portfolio value relative to a lower Downward Threshold. Subsequent positive market returns may then trigger an increased withdrawal rate, and lower remaining portfolio than a lower Downward Threshold causes. A significant market decline at the end of life may then cause ruin. Such is the unfortunate case of retiree 1551. The situation is rare, but warns of the uncertain benefits of rigid planning rules.

Living Within One's Means: Reducing the Minimum Withdrawal Rate

The reason for ruin is obviously attempting to draw more from the portfolio than returns permit. Table 5 demonstrates that less than 2% of retirees fall below a 3% average withdrawal rate and almost no retirees (.14%) fall below 2%. Reducing the Minimum Withdrawal Rate from 3% to 2% should allow more retirees to survive ruin, albeit at low withdrawal rates.

When the Minimum Withdrawal Rate is reduced from 3% to 2% there is little impact on the average withdrawal rate and ending portfolio. See Table 8. However, the probability of ruin is greatly reduced both at a Downward Threshold of 1.0 and at 1.5. This is because the 2% Minimum Withdrawal Rate specifically addresses the needs of those retirees unfortunate enough to experience very low portfolio returns. This group of retirees with underperforming portfolios literally has the choice of maintaining a higher withdrawal rate until they finally reach ruin or living within their smaller means and largely avoiding ruin. It is an unfortunate choice. At a Downward Threshold of 1.5 the risk of ruin can be reduced to .1% if retirees are willing to accept a .41% Minimum Withdrawal Rate, while at a Downward Threshold of 1.0 even a .01% Minimum Withdrawal Rate only reduces the probability of ruin to .26%. These withdrawal rates compare unfavorably to the 4.08% SAFEMAX rate reported in Bengen (1997) due to differences in methodology (in particular Bengen's 30-year planning horizon) and the time period studied. The problem of needing to satisfy the most extreme outlier is dealt with in this paper by defining the SAFEMAX rate as having a probability of ruin of .1%, i.e. the lowest 50 of 50,000 results have been ignored. The argument for this approach is discussed in the unpublished paper referred to earlier and available at http://ssrn.com/abstract=1371695. The SAFEMAX fixed rate based on 1926-2008 data and 50,000 iterations is 1.96% if calculated for age 108 and 1.97% for age 100. The .01% difference in the SAFEMAX rate is a small price for potentially 8 years of additional protection.

Downward Threshold and Minimum Withdrawal Rate	Initial Withdrawal Rate	Average Withdrawal Rate Mean (Std Dev)	Average Ending Portfolio Mean (Std Dev)	Probability Of Ruin Age 108 (Age 100)	Year of First Ruin (Max Years of Ruin)
1.0 3% MIN	5.00%	6.47% (3.04%)	.86 (1.05)	10.31 (9.65)	13 (29)
1.0 2% MIN	5.00%	6.46% (3.05%)	.86 (1.05)	6.67 (5.96)	17 (22)
1.5 3% MIN	4.82%	6.34% (3.12%)	.97 (1.05)	4.01 (3.43)	13 (26)
1.5 2% MIN	4.82%	6.33% (3.13%)	.98 (1.04)	1.60 (1.07)	19 (10)

Table 8
Effect of 2% Minimum Withdrawal Rate on average withdrawal rate, ending portfolio, and ruin
40% Maximum Withdrawal Rate, 2.734 Upward Threshold, .312 Upward Adjustment Rate, and 1.0 Downward Adjustment Rate

The lower Minimum Withdrawal Rate also lengthens the time until the first retiree ruins (from 13 to 19 years at a Downward Threshold of 1.5) and shortens the maximum years of ruin (from 26 to 10 years again at 1.5). With little cost in terms of average withdrawal rate, median withdrawal rate (not shown) and ending portfolio, the likelihood of ruin has been greatly reduced and its onset delayed.

Continuing with the micro effects, Table 9 (an extension of Table 5) compares the percentage of retirees who fail to reach various average withdrawal rates. At a Downward Threshold of 1.0 there is a slight increase in the percent not reaching 5% and 4%, and a decrease in the percent failing to reach 3% and 2%. The lower Minimum Withdrawal Rate allowed some retirees to reduce their withdrawals, avoiding ruin, but did a better job than a 3% Minimum Withdrawal Rate at keeping retirees above a 3% average withdrawal rate. At a Downward Threshold of 1.5 there is an increase in retirees failing to reach rates above 3% and close to zero (.08%) retirees below a 2% average withdrawal rate.

Downward Threshold	Withdrawal Rate			
	5%	4%	3%	2%
1.0 3% MIN	49.30	8.69	2.02	.14
1.0 2% MIN	49.31	9.34	1.89	.11
1.5 3% MIN	45.16	15.35	1.52	.14
1.5 2% MIN	45.15	15.46	2.61	.08

Table 9
Effect of Minimum Withdrawal Rate on percent of retirees not reaching various withdrawal rates
40% Maximum Withdrawal Rate, 2.734 Upward Threshold, .312 Upward Adjustment Rate, and 1.0 Downward Adjustment Rate

Table 10, an extension of Table 4, demonstrates that while the medians are unchanged, the retirees at the very bottom of the distribution are actually helped by the lower Minimum Withdrawal Rate. The average withdrawal rate for the lowest .1% of the distribution rises for both Downward Threshold of 1.0 and 1.5 as those retirees are prevented from ruin for a longer period of time, although probably not entirely. At the lower Downward Threshold even the bottom 2% of retirees are helped.

Downward Threshold	Median	Lowest x% of distribution					
		5%	4%	3%	2%	1%	.1%
1.0 3% MIN	5.15	3.63	3.48	3.28	2.95	2.63	1.92
1.0 2% MIN	5.15	3.55	3.41	3.25	3.03	2.72	1.98
1.5 3% MIN	5.31	3.41	3.33	3.25	3.15	2.72	1.93
1.5 2% MIN	5.31	3.31	3.19	3.06	2.89	2.66	2.06

Table 10
Effect of Minimum Withdrawal Rate on average withdrawal rate of lowest (x) percent of retirees
40% Maximum Withdrawal Rate, 2.734 Upward Threshold, .312 Upward Adjustment Rate, and 1.0 Downward Adjustment Rate

Comparison to Blanchett and Frank (2009)

Blanchett and Frank (2009) is more difficult to compare to due to the differences in controls, their not directly incorporating mortality, and the difference in the return sampling process (normal distribution versus drawing from actual historic events). Blanchett and Frank, like Stout, does not include 2008 data which will understate their probabilities of ruin. Their 4% Initial Withdrawal Rate results in a 2.65% probability of ruin with an average withdrawal rate of 8.83% in the final year of the 30-year planning horizon. For a 4.02% Initial Withdrawal Rate (Downward Threshold of 1.8), this research finds a 2.25% probability of ruin at age 100 (40 year planning horizon) but only a 1.35% probability of ruin at the end of year 30 and a 12.88% average withdrawal rate also in year 30. Despite the differences in methods, the proactive model in this paper appears to provide more return with less risk.

7. Conclusion

Results presented in this paper demonstrate that a Downward Threshold greater than 1.0 can significantly reduce the probability of ruin. For example, the 5.88% probability of ruin for a 60-year old retiree, as reported by Stout (2008), when updated to 2008 data becomes a 9.65% probability of ruin. However, by increasing the Downward Threshold to 1.5 the probability of ruin to age 100 can be reduced to 3.43% with little reduction in the average withdrawal rate and actually an increase in the typical (median) withdrawal rate.

When the simulation is extended to age 108, based on an extrapolation of the Life Table, the probability of ruin is increased, in the above case to 10.31%. To the extent that clients are concerned about the risk of superannuating, practitioners may want to consider methods, such as annuitizing, to limit that risk. The SAFEMAX rate is only .01% lower (1.96% versus 1.97%) when protecting to age 108 rather than age 100.

Updating data to include 2008 significantly affects results. A 4.33% probability of ruin reported by Stout and Mitchell (2006) based on data thru 2004 becomes an 8.68% probability of ruin using data thru 2008 and similar to the effect on the Stout (2008) results. Researchers should be wary of comparing results from studies which utilize different time periods. Likewise, practitioners should be wary of creating client expectations based on historic distributions of security returns.

Micro effects shed additional light on the choice of withdrawal rate strategies. The high positive skewness and excess kurtosis, which result from the high Maximum Withdrawal Rates necessary for many retirees to avoid excess accumulations, make reporting of the mean withdrawal rates misleading. Median withdrawal rates, which are not commonly reported in the literature, may be a more effective way of communicating expectations to clients.

Study of the bottom portion of the distribution of withdrawal rates also helps gain an understanding of how changes in withdrawal rate strategies affect those retirees most likely to ruin. Averages are useful, but as found in this research, may lead to incorrect conclusions as to which withdrawal rate strategy will best help the largest number of retirees.

Further research could focus on acceptable risk levels among retirees; without which researchers are unable to recommend a particular withdrawal strategy. There is also a need for studying optimal solutions, similar to Stout, 2008, for achieving specific (and hopefully acceptable) risk levels including new management techniques; particularly techniques that focus on underperforming portfolios. Studies of standards for effectively utilizing simulation and increasing the comparability of results across studies would also be useful.

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