HOW RISKY IS YOUR RETIREMENT INCOME RISK MODEL?

November 9, 2015*

Name: Patrick J. Collins, Ph.D., CLU, CFA
Phone: 415-291-3002
Email: patrick@schultzcollins.com
Affiliation: School of Management, University of San Francisco

Name: Huy Lam, CFA
Phone: 415-354-3704
Email: huy@schultzcollins.com
Affiliation: Schultz Collins, Inc.

Name: Josh Stampfli, MA EESOR
Phone: 917-348-7910
Email: josh@joshstampfli.com
Affiliation: N/A

Abstract

Adequately sustaining lifetime income is a critical portfolio objective for retired investors. This article provides a brief review of various retirement income modeling approaches including historical back testing, Monte Carlo simulations, and other risk modeling techniques. Implausible assumptions underlying risk models may mislead investors concerning the risk and return expectations of their investment strategies. We compare risk models, evaluate their credibility, and demonstrate how overly-simplified models may distort the risks retired investors face. Failure rate differences are stark: 4% at the low end versus 49% at the high end. The article ends with general comments regarding model risk and practitioner investment advice.

Key Words: The 4% rule, Monte Carlo simulation, Portfolio Sustainability, Retirement Income, Risk Modeling

---

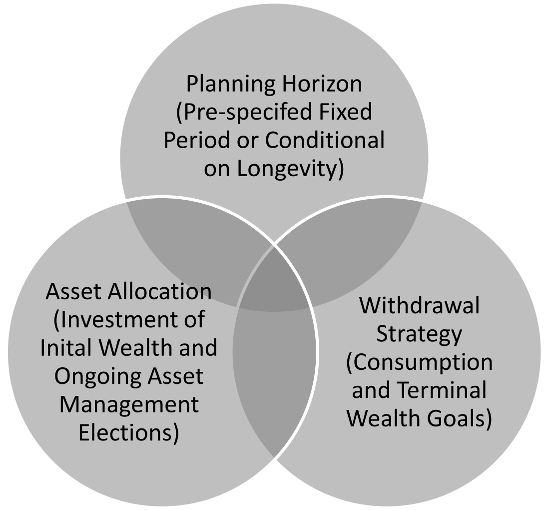
* This article has been accepted for publication in a forthcoming issue of the Financial Services Review. We are grateful to an anonymous referee for a series of helpful comments.

1. Introduction

Sustainability of adequate lifetime income is a critical portfolio objective for retired investors. Commentators often define sustainability in terms of (1) a portfolio's ability to continue to make withdrawals throughout the applicable planning horizon, or (2) a portfolio's ability to fund a minimum level of target income at every interval during the planning horizon. The first approach focuses on the likelihood of ending with positive wealth, or, if wealth is depleted prior to the end of the planning horizon, on the magnitude and duration of the shortfall; the second focuses on the likelihood of consistently meeting all period-by-period minimum cash flow requirements.

Risk models help advisors assess a portfolio's ability to provide adequate cash flow throughout retirement. Conclusions about cash flow sustainability are usually reached by determining the likelihood that withdrawals (fixed amounts, percentage of corpus, or "dynamic") can be maintained for either deterministic or stochastic time periods under various asset allocations and longevity assumptions. Expressed in terms of a Venn diagram, portfolio success lies at the intersection of the three elements in Figure 1.

Figure 1



'Sustainability' differs from the concept of 'feasibility.' Feasibility depends on an actuarial calculation to determine if a retirement income portfolio is technically solvent—current market value of assets equals or exceeds the stochastic present value of the cash-flow liabilities. If the current market value of assets is less than the cost of a lifetime annuity, the targeted periodic withdrawals exceed the resources available to fund them. In short, the portfolio violates the feasibility condition. Determination of the feasibility of retirement income objectives is not subject to model risk because the determination rests on current observables—annuity cost vs. asset value—rather than on projections of financial asset evolutions and the distribution of longevity. Although it is important to track both risk metrics—sustainability and feasibility—as part of prudent portfolio surveillance and monitoring, the remainder of this article focuses on the sustainability / shortfall probability risk metric.

2. Sources of retirement income model risk

Probability assessments are only as good as the models upon which they are based—that is to say, assessments are prone to 'model risk.' In general, model risk arises from several sources:

1. Variables of Interest: Projected model outcomes may differ considerably depending on the range of input variables. Health shocks, inheritance expectations, life insurance availability, and other variables may or may not improve the calculated probability of a successful retirement investment and consumption strategy.
2. Model Sensitivity to Changes in the Value of Input Variables: Output can be notoriously sensitive to small changes in input values; likewise, compounding over long planning horizons can produce large differences in outcome values and likelihoods given small changes in input values.
3. Model Structure: Deterministic inputs will likely project outcomes different than those generated by a model that treats investment, inflation, and longevity variables stochastically. The nature of the model's covariance matrix may be an additional source of estimation error.
4. Model Assumptions: The choice of utility function can influence model output. For example, assumption of Constant Relative Risk Aversion may rank outcomes differently from those flowing from models assuming a Hyperbolic Risk Aversion function.[1] Likewise, the functional form chosen to generate inflation or investment returns will often influence investment recommendations.

[1] Technically, CRRA is nested in hyperbolic risk aversion functions. The critical distinction is between relative and absolute risk aversion. Hyperbolic risk aversion functions [HARA] encompass decreasing absolute risk aversion [DARA]—as wealth increases the investor is more comfortable committing dollars to the risky asset; increasing absolute risk aversion [IARA]—as wealth increases the investor pulls back on the number of dollars put at risk; and constant absolute risk aversion CARA]—as wealth increases the investor keeps the same amount of dollars at risk. Whenever the applicable risk metric defines the percentage of wealth put at risk, the risk model incorporates a *relative* risk aversion measure; whenever the applicable risk metric defines the level of dollar wealth put at risk, the risk model incorporates an *absolute* risk aversion measure. It is, however, the rare investor who remains indifferent to changes in the level of wealth when evaluating investment and spending strategies. This is a central criticism of incorporating relative risk aversion into a retirement income model.

Econometricians often discuss model risk in terms of specification error. Errors may arise as a result of including irrelevant variables in the model, failure to incorporate relevant variables, and inaccurate estimation of input variable values. Specification errors may account for different models producing dissimilar outputs when considering the same problem. This is an underlying reason why any single retirement income risk model may be unable to provide a good assessment of retirement risk.[2]

This article focuses on post-retirement shortfall risk from assessments derived from modeling investment, longevity and inflation related risks. Model-based probability assessments rely, in part, on outputs generated by computer algorithms that approximate, with varying degrees of accuracy, the processes that drive financial asset price changes. Thus, any assessment of the sustainability of a retirement income investment program should not over rely on outputs produced by a single risk model; and, when using model outputs to monitor the portfolio, practitioners should take care to ascertain that the model is academically defensible.

Beyond the above-listed sources of model risk are two other considerations:

1. Bonini's Paradox: models that explain the workings of complex systems are seemingly impossible to construct. As a model of a complex system becomes more complete, it becomes less understandable; for it to be more understandable it must be less complete and therefore less accurate. When a model becomes accurate, it is just as difficult to understand as the real-world processes it represents.

[2] The Society of Actuaries and The Actuarial Foundation's review of a cross-section of financial planning software determined that "…programs vary considerably regarding when the user runs out of assets, if at all. Because of this finding, the study recommends that people run multiple programs, use multiple scenarios within programs, and rerun the programs every few years to reassess their financial position" (Turner & Witte, 2009).

2. Ambiguity in how the investor should preference rank heterogeneous outcomes: outcomes from equally credible models may differ significantly even when each model uses identical inputs and input values. Interpretation of calculated results becomes difficult and there is no clear "winning strategy" or preferred solution path.

3. Modeling approaches

There are a number of modeling approaches to ascertain the likelihood that a portfolio's investment strategy is suitable to its cash flow requirements—i.e., estimating the probability that a jointly determined asset allocation / retirement spending strategy is sustainable throughout the planning horizon absent significant, and possibly difficult to implement, mid-course corrections:

- Analytic formulae (closed form solutions usually within a life-cycle model context)
- Historical back testing of empirical returns
- Bootstrapping (reshuffled historical returns)
- Monte Carlo simulation (assuming a two-parameter normal or lognormal distribution)
- Simulating non-normal distributions (student's t, Pareto, truncated Levy flight, gamma, logistic, exponential, etc.)
- Vector autoregression
- Regime-switching simulation models.

A brief discussion of each method follows.

3.1 Analytic formula

The analytic formulae approach attempts to solve the sustainability question by (1) describing retirement planning as complex systems of equations, (2) transforming descriptive formulae with algebraic manipulation to achieve closed formed solutions, and (3) plugging in assumed values for the independent variables to arrive at a final conclusion. Most formulaic solutions pile assumption upon assumption regarding the functional forms and parameterized values of the numerous variables included in the model. In order to make the mathematics tractable, many models assume that stock returns are independent, identically distributed, and, as a consequence, that the underlying distribution of stock returns is stable. Input variables may include rates of returns and volatilities for financial assets, inflation rate behavior, interest rate term structure, and the form of an investor's utility function. Specific input variables are often estimated using econometric techniques which, although critically important to an assessment of a model's credibility, are, nevertheless, tangential to the focus of this article.[3] Generally speaking, most studies of stock price changes reject the hypothesis that the return series is normally distributed, with the most often cited deficiency as a failure to capture the 'volatility of volatility.' [4]

Finding the closed formed solutions to analytic formula models is a daunting task that often requires applying highly sophisticated integral calculus or solving intricate partial differential equations.[5] Huang, Milevsky, and Wang (2004), for example, use a formulaic approach to conclude that an inflation adjusted withdrawal rate equal to 3.33% of a 65-year-old investor's starting portfolio value exhibits a 95% sustainability rate.

---

[3] An excellent review of econometric issues in the modeling of asset price returns is Carol Alexander's (2008) four volume series Market Risk Analysis.

[4] See, for example, Karolyi, G. Andrew (2001) "Why Stock Return Volatility Really Matters," Strategic Investor Relations.

[5] Analytic models often assume lognormality when returns are measured in discrete time; when returns are measured in continuous time, the models often assume that returns follow a geometric Brownian motion process. Many analytical models must incorporate geometric Brownian motion as a pre-condition to using the techniques of integral calculus. Over the small intervals serving as units of time for continuous finance models, return differences between normal and non-normal distributions are minimal and seemingly inconsequential. Aggregation of results over time often relies on the Central Limit theorem's tenet that the mean of a sequence of normal random variables is, itself, a normal random variable that, at the limit, exhibits the mathematical property of convergence. However, for longer term planning horizons, assuming normality in the distribution of investment returns may have severe economic consequences if returns are, in fact, not normally distributed. Additionally, the Central Limit theorem characterizes the distribution of sample means and provides only limited insight into the value of the variance statistic.

Despite its mathematical elegance, application of the analytic formula approach to sustainability analysis has been quite limited. Solutions often seem enigmatic, and generally require a complex array of equations. Another weakness of using analytic formulas to arrive at retirement success rates lies in the fact that analytic models often ignore randomness in the independent variables. Asset returns are the prime example. For instance, in order to assert that an individual willing to run out of money after ten years can withdrawal nearly four times as much as one determined to preserve capital forever, Orszag (2002) assumes a constant dollar return of 3% in his equations. Returns, however, aren't constant; and the sequence of returns can play a major role in portfolio depletion rates.

3.2 Historical back testing

Perhaps the simplest approach for determining the sustainability rate of a retirement income plan is Historical Back Testing (also known as Rolling Period Analysis or Overlapping Period Analysis). As the name implies, this approach relies on a sufficiently long set of historical returns data. The historical returns used are the actual returns an investor's portfolio would have experienced, given its asset allocation. Many retirement income risk models specify a withdrawal strategy throughout the planning horizon–often a fixed 20, 25 or 30 years. A commonly evaluated strategy is the 4% rule which withdrawals an annual inflation-adjusted amount equal to 4% of the portfolio's initial dollar value. The historical back testing method tests the success or failure of the retirement plan for each unique planning horizon in the data set of historical returns. The number of unique periods is determined by rolling up the start date of each planning horizon by a single increment of time. For example, Bierwirth (1994) begins his analysis in 1926 and uses a one year rolling window to calculate 42 unique 27-year rolling periods, ending in 1992. Each sample period is "unique" by virtue of the fact that its start year drops out of the data set as a new ending year enters the data set. Intervening years, however, continue to appear in multiple samples. Assuming that the past is indicative of the future, the historical model calculates the likelihood of retirement income sustainability by dividing the number of successful planning periods by the total number of rolling periods for any given asset allocation / retirement spending strategy combination. The combination with the highest success rate is considered optimal when optimality is measured by the likelihood that the unmodified or 'autopilot' spending strategy is sustainable over the applicable horizon.

The acceptable retirement income sustainability rate is highly subjective, and depends on investor circumstances and risk tolerance. However, for the purposes of this discussion, we will benchmark model outputs relative to the 75% guideline of Cooley, Hubbard, and Walz (2011). That is to say, a retirement risk model incorporating an asset management strategy exhibiting a 75% or greater likelihood of success is acceptable to a retired investor.[6] The key question is: how credible is the success probability derived from a particular risk model; or, when is a model's 75% or greater success rate not really indicative of a 75% or greater likelihood for success?

Historical back testing is easy to understand, and is a simple way to calculate relatively accurate assessments of what would have happened.[7] However, an investor relying on such an approach should proceed with caution. Decisions based solely on historical data force an investor to have faith in the highly dubious assumption that future returns will mimic realized past returns.[8]

---

[6] See DiCarlo Jr. and Fast (2008) for a survey of opinions found in financial advice literature regarding the acceptability of various levels of portfolio shortfall risk. The article focuses primarily on standards of prudence for management of trust-owned investment portfolios.
[7] We note additional complications surrounding continuous index availability throughout the planning period as well as portfolio drift in the absence of constant rebalancing to the designated asset allocation target.
[8] McGoun (1995) argues that the empirical distribution of financial asset price returns is not a measure of risk. It is merely a measure of historical realizations which may or may not be applicable to the current economic situation.

Furthermore, as stated, the rolling period method over-weights observations in the middle of the time period relative to observations occurring at the beginning or end. Such over weighting creates statistical bias. Extreme observations found in over-weighted middle time horizons can cause clusters of failed sustainability. O'Flinn and Schirripa (2010) attribute one such cluster of failures in their study of withdrawal plans to significant inflation during the 1970s and 1980s. Although modeling the serial and cross correlation of asset returns is desirable, the rolling period approach fails to provide a sufficient number of independent samples for credible portfolio sustainability testing. A looping method is one way to deemphasize the importance of the middle observations in the data set (Davis, Wicas, & Kinniry, 2004). Instead of stopping at the last observation date, the looping method carries calculations back to the beginning of the historical return sequence. However, the looping method distorts the value of the autocorrelation statistic in the historical dataset and presents its own statistical difficulties. Although interesting, the pure history model fails to provide assurance that past conditions are sufficiently similar to current conditions so that they act as conditions precedent.

3.3 Bootstrapping

Retirement income risk models sometimes employ a bootstrap approach to develop a broader set of financial asset returns. Bootstrapping is a process that develops return sequences by randomly drawing, typically with replacement, from the historically realized set of returns. Randomly drawn sequences serve as possible future economic paths for testing the sustainability of spending policies. A large number of economic paths can be bootstrapped, thus providing a

---

McGoun's article presents a history of risk measurement by economists. It provides a good theoretical basis for a monitoring and surveillance system using current observables to supplement shortfall risk measures.

larger set of scenarios for testing sustainability than is possible with the historical back testing process.

Bootstrapping, depending on the structure of the risk model, can either preserve correlations across asset classes—by making random draws which take a period's realized returns across two or more asset classes ("cross-sectional" random draws)—or eliminate correlations across asset classes by taking random draws of asset returns from differing periods. It is beneficial to preserve the covariance of asset class returns; and so studies based on bootstrapping often preserve cross-sectional correlations. Spitzer (2008), for example, uses the bootstrapping method to discern the best withdrawal rate and asset allocation over a pre-specified time horizon given an acceptable portfolio sustainability rate. However, unlike historical back testing, the bootstrapped scenarios do not preserve serial correlations evident in empirical returns, since the bootstrapped returns for each time period are independent draws from the set of historical outcomes.

Much like the rolling period technique, bootstrapping requires a long history of asset returns. Without a long history, the sequences created from a small set of possible outcomes will be too similar, with the result that the retirement income risk model performs sustainability tests on scenarios that do not credibly reflect potential future economies. Even with a large history, sampled outcomes cannot differ from the pre-specified set of observables; therefore, propagated series over rely on the past in order to predict the future.

3.4 Monte Carlo simulation: normal distribution

The Monte Carlo method further expands the set of possible outcomes for sustainability testing. It overcomes the limitation of relying on realized past returns as the basis of potential outcomes inherent in both the historical back testing and bootstrapping routines. Monte Carlo simulators generate sequences of potential economic paths by drawing random samples from probability density functions meant to represent the true underlying distribution of financial asset returns. This allows for a much greater range of potential outcomes. Most commonly used Monte Carlo simulation engines assume asset class returns adhere to the well-known, bell-shaped normal, or log-normal, probability distribution which is often parameterized by the historical mean and variance. Furthermore, a well behaved correlation matrix is used to preserve cross correlation in simulated outcomes. Using a simple two asset class Monte Carlo simulation model with log-normally distributed returns, Klinger (2011) reports sustainability rates above 85% for all retirement strategies he analyzed.

Klinger's results, however, are based on the assumptions that stock returns average 6.9% per annum with a standard deviation of 15.7%, and that bonds return 6.6% on average with a standard deviation of 2.4%. The values of asset return distribution parameters in simple Monte Carlo simulation models have been a point of contention.[9] Blanchett and Blanchett (2008) point out that sustainability rates derived from simple Monte Carlo simulations are very sensitive to changes in the assumed expected rates of returns and standard deviations. However, the debate over what values constitute the most reliable parameters for the mean and standard deviation may be a secondary concern if asset price returns are not normally distributed. Moreover, much like the bootstrapping method, simple Monte Carlo simulations destroy serial correlations evident in historical asset price returns. As we shall see, more complex simulation engines can imitate the autocorrelated nature of returns and the time-varying behavior of risk; but for now, we turn our attention to the topic of non-normally distributed returns.

---

[9] Milevsky and Abaimova (2006) observe that different commercially-available Monte Carol simulation programs produce different solutions even when given the same inputs.

3.5 Monte Carlo simulation: non-normal distributions

The normality assumption implies that returns are stationary, symmetric and, at reasonable values for the standard deviation statistic, have low probabilities of realizing extreme deviations from the mean. Simulated returns based on the Gaussian distribution exhibit, on average, neither skewness nor excess kurtosis. Statistical analysis of historical returns by Lee (2009), however, indicate that realized returns are slightly skewed (asymmetric) and have higher likelihood of extreme events than predicted by a normal distribution (leptokurtic, fat-tailed, or heavy-tailed). But if the bell curve does not accurately represent the true underlying distribution of returns, what other stable distributions can Monte Carlo engines employ? Levy and Duchin (2004) fit monthly historical asset returns to eleven different probability distributions and infer that the logistic distribution is the best fit for many asset classes. Athavale & Goebel (2011) simulate asset returns based on ten different distributions (Beta, Extreme, Gamma, Laplace, Logistic, Lognormal, Pert, Rayleigh, Wakeby, and Weibull) to test the 4% withdrawal rule. They conclude that a 4% withdrawal rate tested in non-normal distributions generally results in a lower sustainability rate when compared to test results using a simple Monte Carlo method that assumes distributional normality. The major econometric weakness of simulating one or more stable distributions, however, is that each distribution assumes that periodic returns are independent and stationary with the result that the model fails to capture autocorrelation.

3.6 Vector autoregression

Simulations using non-normal distributions address some problematic assumptions; but such simulations fail to account for autocorrelation in asset price returns. The time-dependent nature of asset prices manifests itself through momentum and mean reversion in the return series. More complex simulations based on vector autoregressive processes, however, can better reflect serial correlations. Pang and Warshawsky (2009), for example, use a Vector Autoregressive simulation model to compare six retirement plans employing combinations of mutual funds and annuities. In addition to addressing the serial correlation issue, simulations based on vector autoregressive models can also incorporate what Campbell and Viceira (2005) call "state variables", which are variables useful for forecasting asset returns. However, the vector autoregressive approach is often complex and, for investment advisors, difficult to implement. Furthermore, the coefficients of the vector autoregressive equation must be estimated, and adding numbers of economic variables significantly reduces the precision of estimated parameters (Campbell & Viceira, 2005). With additional suitable but complex extensions, a vector autoregressive model can model heteroskedasticity and, therefore, capture a portfolio's time-varying risk.[10]

3.7 Regime switching

An alternative to complex vector autoregressive conditional heteroskedastic models is a regime switching model.[11] Regime switching models assume returns come from two, or more, sets of probability distributions—one representing asset price behavior during states of normalcy, and the other(s) during financial crises. When financial crises occur, markets are afflicted with a

---

[10] Research by Kopcke, Webb, Hurwitz, and Li (2013) compares outputs generated by three Vector Autoregressive models.

[11] For an introduction to Markov chain transition probability matrices, see Laverty, W.H., Miket, M.J., and Kelly, I.W., "Simulation of Hidden Markov Models with Excel," Journal of the Royal Statistical Society: Series D (The Statistician), Vol. 51, no. 1 (March, 2002), pp. 31-40. Hybrid models utilize a regime switching mechanism in conjunction with a vector autoregressive conditional heteroskedastic process. For example, a variety of autoregressive processes are incorporated into regime switching models by Hamilton and Susmel (1994). Litzenberger and Modest (2010) develop a model with eight different states in which the financial asset behavior within each state is modeled by a normal distribution with state-dependent means and standard deviations. Ameriks, Caplin, and Van Nieuwerburgh (2008) utilize a four-state health model to examine issues in retirement income planning which seeks to preserve a threshold income level. The matrix is a Markov chain transition matrix with an age-varying, one-period" transition probability. The evolution of health status is also an important variable in the model described by Gupta and Li (2013).

flight to liquidity and with the contagion of fear. Expected returns fall, volatility increases, and correlations converge towards one.[12] The model we present in this paper generates asset returns from two separate market regimes [Bull and Bear], with inflation modeled as an autoregressive process.

Assuming a two-state model, for each simulated return path, the regime switching algorithm calculates the probability either that the underlying economy will transition to the other regime in the next period, or a corresponding probability (1 – probability of switching) that the economy will remain in the same regime. Both Ang and Bekaert (2004), and Kritzman, Page, and Turkington (2012), for example, utilize a variable Markov process with a constant transition probability. As simulated economies evolve through bull and bear markets of various lengths and magnitudes, the Ang and Bekaert model generates the volatility cluttering and correlation breakdowns that characterize asset price behaviors in turbulent market conditions.

As stated, an advantage of a regime switching approach is its ability to capture dynamic correlation and time-varying risk premia. Thus, instead of using average unconditional correlation values determined by the historical data, the approach applies correlation values conditioned on the economic state of nature. For example, over the entire sample period, an asset class may exhibit a mean of 10% and a standard deviation of 20%. However, during bull markets, the parameter values may be +18% mean and 15% standard deviation; while, during bear markets, the parameter values may be -23% and 25% respectively. Thus, simply using the unconditional mean, standard deviation and correlation values for the aggregate historical period cannot capture realistic asset price behavior.

[12] Smith and Gould (2006) discuss the differences between unlucky draws from a stable probability distribution vs. a substantial change (for the worse) in the probability distribution itself.

4. Inflation and model risk

Given the number of modeling approaches, it should not be surprising to find that there is a correspondingly wide range of model outputs. However, the investor, or advisor, may base decision making on the output from only one type of modeling approach; and, furthermore, may not realize that even this single-perspective view of retirement risk may flow from a model that incorporates over-simplified assumptions regarding critical factors such as inflation, investment costs, and rebalance frequency. Here is the critical point: variations in a model's mathematical structure and input assumptions can lead to outputs suggesting drastically different conclusions regarding the suitability of current asset management policies to a client's financial objectives. For example, even within the simplistic Rolling Period Analysis approach, using historical returns spanning different time periods or varying the size of the rolling window can generate substantially different success or failure probabilities. Understanding such sensitivities is essential to discerning the trustworthiness of outputs from a retirement income risk model.

To illustrate model risk under a sustainability risk metric, we present outputs from our proprietary risk modeling system which has the capacity to illustrate simulated outputs under varying asset management and modeling approaches. The initial model incorporates only a few basic variables and reflects a simple bell-curve structure for the distribution of future investment returns. We then utilize risk models that incorporate more variables and that allow for greater modeling flexibility. We demonstrate how an over-simplified model—many of which form the basis for normative articles in the financial press—may seriously distort the risks faced by retired investors.

Consider a simple, annually rebalanced, two asset class portfolio, allocated 70% to U.S. equities and 30% to U.S. bonds. Initially, the model ignores fees, taxes, and transaction costs. The model assumes normally distributed asset returns parameterized by historical averages and standard deviations. Portfolio price evolutions are multivariate normal, where the process derives from a single variance/covariance matrix assuming static (average historical) correlation values for all future economies. The initial portfolio value is $1,250,000 with an annual inflation-adjusted withdrawal of $60,000 for exactly 30 years. The portfolio consists of only two asset classes and, therefore, is not well diversified.

To best illustrate the risk of relying on over-simplistic retirement income risk models, we focus on the inflation variable. A common way to incorporate inflation into a risk model is to assume a constant rate. However, retirement income risk models are hypersensitive to the level of assumed inflation with the result that success or failure probability assessments may differ widely. Table 1 exhibits the inflation adjusted results of our two-asset class model under three different levels of fixed inflation: 3%, 4%, and 5%. A one percentage point difference in the assumed rate generates a marked divergence in ending portfolio values. At the median[13], the model assuming 3% inflation yields an ending value worth roughly a million dollars more in goods and services than the model assuming 4% inflation. In terms of the 75% probability of sustainability benchmark, the outcomes of both the 3% and 4% inflation models are acceptable [sustainability rate = (1 – bankruptcy rate)]. The 3% inflation input yields a 91% sustainability rate, which is fully 9% higher than the 4% inflation input's 82% sustainability rate.

[13] The 50th percentile value of the distribution of outputs when ranking outputs (trials) according to terminal value from low (1st percentile) to high (100th percentile).

Inflation, however, is not constant; and, dealing with inflation in such an oversimplified manner produces implausible outputs. More credible risk models treat inflation as a random variable. Alongside the three constant inflation outputs in Table 1 are two additional outputs, one based on inputting inflation starting at its long term average (4.32% during the period from 1973 to 2012), and the other based on inputting inflation starting at its previous 12 month average (1.74% in 2012). Furthermore, the enhanced risk model generates paths of future inflation by treating it as a stochastic variable exhibiting a mean reversion factor. The inflation process has an expected value (drift) factor, and an innovation (diffusion) factor. The result is an output that accounts for serial correlation in a random but "sticky" time series of inflation rates. Since the 2012 rate is so much lower than its historical average, the stochastic model of inflation using 2012 inflation as its starting point leads to a more plausible estimation of a portfolio's sustainability rate. This result, in turn, contrasts with the results generated by a stochastic process starting off in the historically average inflationary environment.[14]

Table 1

Risk Model with Inflation:	Constant 3% Inflation	Constant 4% Inflation	Constant 5% Inflation	Stochastic Long Term Average	Stochastic Previous 12 Months
Ending Wealth at the 50th Percentile	$2,417,712	$1,347,812	$665,967	$1,050,470	$1,423,391
Ending Wealth at the 30th Percentile	$1,232,951	$499,252	$33,002	$129,020	$397,651
Ending Wealth at the 10th Percentile	$71,029	$0	$0	$0	$0
Bankruptcy	9%	18%	29%	26%	21%

[14] Our risk model also incorporates CPI into the variance/co-variance matrix of asset classes used to generate asset price evolutions. This means that there is a complex interaction between increases in CPI—which are more probable under a mean-reversionary process when the inflation rate is below its historical average—and asset price evolutions which may be negatively correlated to inflation rate increases.

Assets Ever < $750k Inflation Adjusted	27%	43%	57%	51%	44%

We direct the interested reader to the 2006 article by Paul Kaplan (2006) for additional discussion of how success probabilities differ depending on the retirement income risk model's inflation assumptions. Kaplan models inflation in three ways: constant, single-period lagged autocorrelated process, and two-period lagged autocorrelated process.

5. Illustrating risk:

5.1 A simple two-asset class model

Having illustrated how five distinct inflation behaviors yield significantly different assessments of portfolio sustainability, we now turn our attention to the assumptions underlying the modeling of asset price evolutions. In our earlier discussion, we introduced the Monte Carlo method for generating the distribution of future asset prices. Risk models using simple Monte Carlo simulations assume normally distributed asset returns parameterized with historic means and standard deviations. The simple Monte Carlo method also utilizes a historical correlation matrix to account for the co-movement of asset prices. We designate this model the "NH" model, for "Normal Historical". It is a model commonly used by financial advisors.[15]

Next, we modify the historical parameters of the Basic Monte Carlo model to conform to the single-index, Capital Asset Pricing Model's assumption that all investments have the same long-term expected real Sharpe Ratio in efficient markets. This modification assumes that expected returns plot on the capital market line. However, the model maintains the assumption of

[15] Since the realized path of history is a single vector of results—a sample of one—the NH model produces a credible distribution of investment outcomes only under the assumption that future investment conditions will mirror previous economic environments.

distributional normality. We designate this model NE, for "Normal Efficient". Although the NH and NE models differ in the values they use to parameterize the normal distribution assumption, both assume time-invariant parameters.

Finally, we consider modelling variations that mitigate many statistical difficulties arising when assuming distributional normality. We previously demonstrated that it is possible to originate economic evolutions from non-normal distributions using a number of techniques: bootstrapping, Monte Carlo simulations sampling from non-normal distributions, vector autoregressive models, and regime switching simulations. Despite the fact that each approach has its pros and cons, we prefer the regime switching approach when cumulating dollar values over long planning horizons. Mary Hardy (2003), a prominent Canadian actuary, stresses the importance of using credible risk models when cumulating dollar values over lengthy planning horizons. She provides strong support for using a regime-switching model. Following an in-depth survey of various modeling tools and techniques, she concludes that the best way to approximate the range of future portfolio dollar value within an asset/liability matching context is through a two state regime-switching lognormal model.

Unlike simple Monte Carlo simulations, return series produced in a regime switching engine exhibit all of the following empirical asset price behaviors: skewness, fat-tails, autocorrelation, volatility clustering, and dynamic correlations. The first of our regime switching models assumes an investor with an agnostic view of the capital markets. This means that there is no attempt to predict whether the immediately forthcoming returns will start in either a bull market or a bear market. Outputs, therefore, do not depend on the accuracy of the investor's forecasting ability. This variation of the risk model randomly selects the underlying initial state of the economy where the selection of an initial bull or bear market state reflects historical relative frequency. We identify this randomly selected initial-state model "BB" for Bull/Bear.

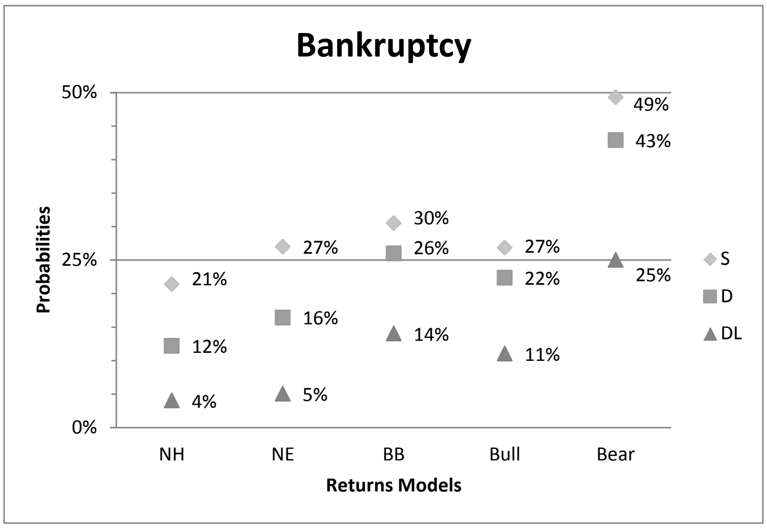
However, if the investor wishes to impose a market viewpoint, there is an opportunity to specify the forthcoming initial underlying economic state from which financial asset returns are generated. For example, if the investor strongly believes that asset prices will rise in the next period, we start our regime switching procedure in a bull market, and we identify the non-randomly selected initial-state model as the "Bull" model. On the other hand, if the investor wishes to reflect a pessimistic outlook for forthcoming returns; or, if the investor simply wishes to see how starting retirement in adverse economic conditions impacts portfolio values, then we start the return-generating process in a bear market. We identify this non-randomly selected initial-state model as the "Bear" model.

Putting these models into the context of retirement planning, we generate return evolutions for an annually-rebalanced, two-asset class portfolio, without accounting for taxes, fees and other investment costs. Inflation follows a stochastic process using a trailing 12 month average starting value. We present the inflation-adjusted outputs generated for this "simple" portfolio under the five distinct asset price models introduced above:

1. Multivariate Normal Distribution Historical Model (NH)
2. Multivariate Normal Efficient Returns Model (NE)
3. Regime Switching Model Market Agnostic (BB)
4. Regime Switching Model Bull Market Prediction (Bull)
5. Regime Switching Model Bear Market Prediction (Bear)

The risk-metric of interest is the likelihood that a $1.25 million portfolio withdrawing an inflation-adjusted $60,000 annually (4.8% of its initial value) will become fully depleted—i.e., bankrupt—prior to the end of the planning horizon.[16] In Figure 2, we designate the series of model outputs by the symbol "S", for their standard, two asset class allocation. Figure 2 also presents outputs from models we'll discuss later.

Figure 2



Projected portfolio sustainability rates for the five models of the S series clearly depict a wide range of possible outcomes. Although the commonly used NH model's bankruptcy rates are less than 25%, the investor is left to ponder the extent to which this favorable picture is merely an artifact of a risk model that fails to incorporate critical elements of asset pricing behaviors. The NE model modifies parameters fit directly from historical data; it produces a 6% higher portfolio risk profile. The market agnostic retirement income portfolio risk model (BB) indicates that the risk of portfolio depletion by myopically following a fixed asset allocation / 4.8% spending strategy is significantly greater than suggested by the simple Monte Carlo model. Not

[16] In this example, we arbitrarily fix the planning horizon at 30 years. We later expand the example to incorporate the force of mortality.

surprisingly, by starting the simulations off in a predicted Bull market environment, the bankruptcy rate decreases—but only by 3% over the agnostic view of future markets. It is still 6% higher than projected by the simple Monte Carlo model. On the other hand, having a pessimistic view of near term market returns drastically increases portfolio bankruptcy rates to approximately 49%. This outcome is illustrative of return sequence risk faced by retires.[17] If a retiree wishes to impose a market viewpoint on his or her investment strategy—always a dangerous and uncertain proposition; or, if a retiree wishes insight into financial asset performance in "worst-case" economies, the Bear model option illustrates a distribution of future results reflecting implied pessimism.

5.2 A diversified portfolio

Luckily, investors don't live in a two asset class world. The portfolio returns generated in our simplified set of models suffer from unsystematic market risk. A savvy investor would diversify away the unsystematic risk inherent in their portfolio, and move it to, say, a 14 asset class allocation[18]. Doing so allows us to illustrate the effect of diversification within each of the five return-generating models. In Figure 2, we label the new series "D", for diversified. By taking advantage of broad-scope diversification, three of the five models in series D exhibit acceptable sustainability rates, compared to just one in series S. Furthermore, each diversified portfolio in model series D has a lower bankruptcy rate compared to their two asset class counterparts in series S.

---

[17] The risk that the combination of (1) unfavorable portfolio returns realized early in retirement and, (2) portfolio withdrawals will deplete the portfolio's dollar value to the point where future favorable returns operating on smaller dollar values are insufficient to offset the early losses.

[18] The diversified portfolio invests in broad range of domestic and developed foreign equities with a tilt towards value oriented and small capitalization stocks. For fixed income, the portfolio invests in diversified pools of high quality domestic and foreign bonds with short and intermediate maturities. Other holdings in the portfolio include positions in diversified baskets of US REITs (Real Estate Investment Trusts) and emerging market stocks.

However, it's interesting to see the relative benefits of diversification abate in models incorporating regime-shifting bull/bear methodologies.[19] When measured by the improvement in failure rates, there is only a diversification advantage of just 4% in the BB approach versus a 9% improvement in the NH model. The relative diversification benefits diminish in a regime shifting model. Turbulent market conditions tend to raise the correlation between asset classes above their long term historic averages, thus reducing the benefit of diversification. Neither the NH or NE models can replicate such conditions since their returns generating mechanism uses only a single correlation matrix.

5.3 Longevity

Thus far, the model series assumes a fixed 30-year planning horizon. This assumption is not realistic for individual investors with uncertain life spans. A 30-year planning horizon may overstate shortfall risk for many post age 65 retired individuals. To get a more realistic view of risk, the uncertain nature of longevity is integrated into the next series of simulations. We term the series "DL" for a diversified portfolio reflecting an uncertain lifespan. The lengths of simulated trials are no longer preset at 30 years. Rather, the distribution of life span reflects the Society of Actuaries' mortality table for the subpopulation of high-income, white collar retirees—the group most likely to employ the services of a financial advisor. The distribution of lifespan for this population group differs significantly from the distribution of lifespan for the general population. Longevity risk (the likelihood of outliving resources) is a stochastic variable, not simply an average. Inputting Social Security general population mortality data, for example, decreases the failure rate probabilities significantly. Although beyond the scope of this

[19] An early study of the effect of diversification on portfolio sustainability is Collins, Savage, and Stampfli (2000). This article presents results from simulating multi-asset class portfolios consisting of globally diversified stock, bond and real estate investments.

discussion, the issue of expected vs. actual life span is important when modeling retirement income portfolios.[20] One result is that the uncertainty of an investor's remaining lifetime increases with age despite a reduction in expected longevity. In fact, actuarial life expectancy is conditional on attained age—the longer you live, the longer you are expected to live.

The DL model assumes a 68 year old female investor in excellent health. Average life expectancy for such an investor is roughly 19 years (mean = 18.7; median = 19.1); and, therefore, we expect bankruptcy rates to fall. Trial length is the lesser of 360 months (30 years), the date of portfolio depletion (bankruptcy), or the month of death, whichever event comes first. As Figure 2 shows, the mortality-adjusted time horizon lowers bankruptcy risk dramatically. Four of the five returns models incorporating longevity exhibit portfolio sustainability rates in excess of the 75% acceptability benchmark.

An investor using a simple Monte Carlo simulation program anticipates that there is an approximately 4% chance that the investment strategy, operating across time without modifications, will be unable to meet critical needs. An investor with a pessimistic near-term market outlook, anticipates an approximately 25% chance that the retirement portfolio will be unable to provide the target lifetime income. The empirical data underlying each model's output is exactly the same. This means that the differences in risk measurement is due solely to the structure of the retirement income risk models—i.e., model risk. This does not imply that the model generating worst-case results is the most credible. However, it does suggest that

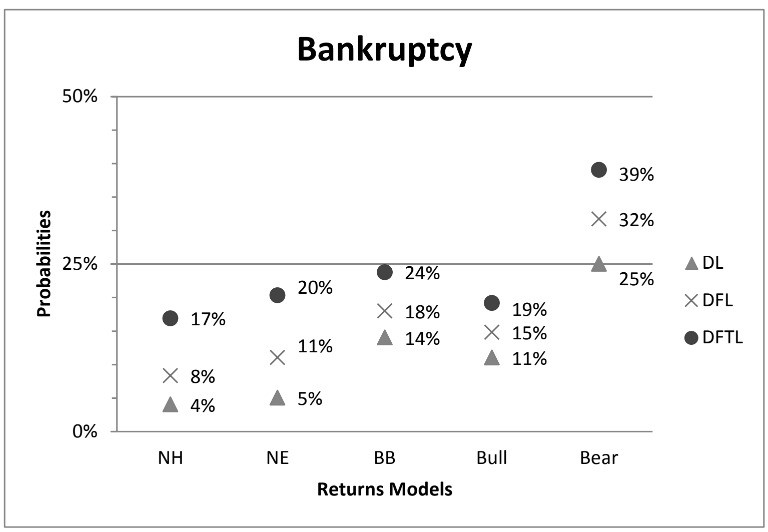
[20] One half of the population will live longer than the average life span; and, in some cases, the individual's life span may be many years above the average. Distributions of life spans approach exponential distributions with long tails. The research of Brown & McDaid (2003), sponsored by the SOA, reviews 45 research papers that examine factors, including pre- and post-retirement income, that affect mortality.

inappropriate investment advice may be offered to investors based on outputs from overly simplistic models.

5.4 Portfolio frictions (fees & taxes)

The simulations presented to this point have ignored investment costs such as trading commissions, custodial/trustee fees, mutual fund / ETF expenses, investment advisor fees, and so forth. The next series of simulations reflect the costs of investing for a diversified portfolio paying management fees[21] and transaction costs. Figure 3 labels the series "DFL"—diversified portfolio paying fees and expenses and reflecting uncertain lifespan. As seen in Figure 3, incorporating fees and transaction costs increases bankruptcy rates.

Figure 3



[21] In place of a simple flat percentage fee, we elect to use the following progressive fee schedule: 1% on the first million, 65 basis points (bps) on the second million, 35 bps on each dollar between two and five-million, and finally 25 bps on any amount above five-million. If the portfolio grows, this laddered fee structure is less detrimental to the portfolio than a flat 1% fee. A minimum fee of $5,000 is charged to the portfolio if it ever falls below half a million dollars. The portfolio is passively managed with index funds and rebalanced annually.

Just as death is certain, so are taxes. Taxes, however, are particularly difficult to model because of (1) myriad nuances within the tax code, and (2) variations in investor tax circumstances. The type of investment account often determines the nature and extent of taxation. Additionally, the assets themselves may be tax exempt; interest and dividend income are often taxed differently than capital gains income; turnover rates may determine whether long or short term capital gains rates apply, and so forth. Nonetheless, taxes are a cost factor either for setting the threshold target budget or for determining the drag on portfolio growth. Usually, investors calculate their income needs by including an estimate of the taxes that they must pay. However, advisors may, from time-to-time, need to incorporate taxes directly into the retirement income model.

Rather than ignore taxes, we have made some simplistic assumptions that allow us to evaluate, to a reasonable extent, the impact of taxes. Our model maintains a constant tax regime in that it does not forecast changes in tax law over the investor's lifetime. We do not include an allocation to municipal bonds, and we assume that no assets are held in tax sheltered accounts such as 401(k)s, 403(b)s, IRAs, etc.[22] Interest income is taxed at the ordinary income rate—assumed to be 20%, and all dividends are tax qualified and are taxed at a long-term rate of 15%. Long-term rates are also used for the taxation of investment gains when assets are sold for both withdrawals and rebalancing. Assets are taxed at a blend of short and long-term rates, depending on the annual turnover rates. We assume that an asset's initial cost basis is half that of value in the start year, and that portfolio investment positions are low cost, low turnover, passive indexed funds.

[22] These assumptions allow us to bypass the issue of asset location. Gordon Pye (2001) discusses at length how a portfolio's asset location affects the withdrawal rate adjustment amount necessary to maintain portfolio sustainability in after tax dollars. Incorporating asset location into a risk model adds another dimension of analysis, and model risk, to retirement planning. Although there are well known rules of "conventional wisdom" that address withdrawal strategies from various accounts, such as "draw down taxable accounts first, then turn to taxed-deferred accounts", Coopersmith and Sumutka (2011) demonstrate the benefits of a tax-efficient optimization approach over conventional wisdom.

The model further assumes that tax loses and or tax payments are accounted for monthly. Both operations are reasonably consistent with the notion of prepaying estimated taxes quarterly.

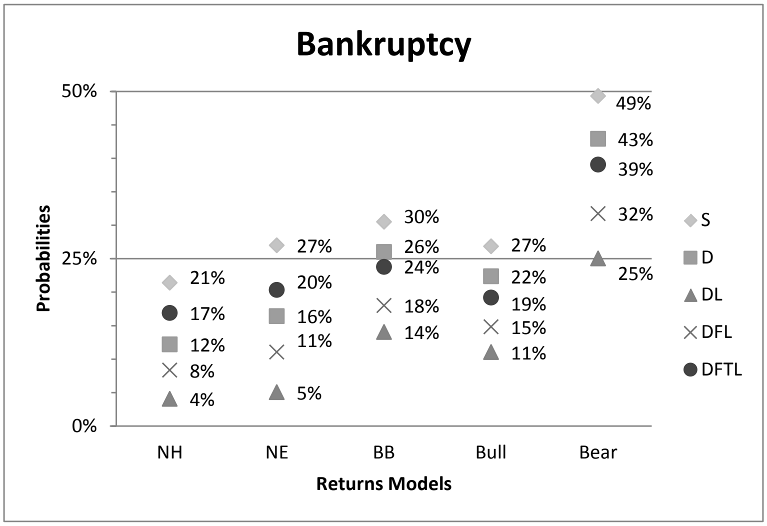
Figure 3 also compares previous pre-tax results to results from a model encompassing taxation. In this case, the investor specifies that she requires a $5,000 minimum monthly income net of income tax liabilities. We label the new outputs "DFTL" for a diversified portfolio paying investment fees, paying taxes, and incorporating an uncertain planning horizon by virtue of investor mortality. Since taxes are an additional cost, the DFTL bankruptcy rate probabilities all plot above the previous outputs that ignored taxes.

The effect of our tax model is quite significant: roughly doubling the failure rates in the normal distribution models, and raising bankruptcy rates from 4 to 7 percentage points in the regime switching models.

5.5 Summary of results

Figure 4 puts all five previously examined series together on a single graph.

Figure 4



Our example started with the investor, holding a simple two-asset class portfolio (the S series), demanding a constant retirement income stream for 30 years. Bankruptcy rates from those simulations were somewhat alarming. The investor then sought help from an investment advisor who recommended diversifying the portfolio. The benefits of diversification are evident in the lower failure rates in the D series. However, diversification is less beneficial in the regime switching models. This result is not surprising given the tendency for correlation values to move towards one in times of market turbulence. However, it is unlikely that a retired investor will need income for exactly 30 years. Incorporating the variability of lifespan into the model yields results presented in the DL series. The drastic fall in failure rates across all risk models indicates the importance of accounting for uncertain lifespan. When unknown lifespan is considered, bankruptcy rates fall to a third of their level in the NH and NE risk models: 12% to 4% and 16% to 5%, respectively. Failure rates are only cut in half in the BB and Bull models:

26% to 14% and 22% to 11%, respectively. The Bear model exhibits the least benefit from the force of mortality: a 42% reduction in shortfall rates from 43% to 25%. The DL series depicts the lowest level of failure rates among model outputs. Mortality affects sustainability rates in the normal-distribution models much more so than it does in the regime switching models.

Investment advice is not free and asset management generates trading cost; therefore, we further expand the models to include investment costs and advisory fees. The DFL series incorporates the force of mortality into a diversified portfolio model decremented for investment costs. The DFL model is a credible retirement income risk model when the investor's budget is defined in pre-tax dollars. However, retirement income may come primarily from a trust established for the benefit of the investor. This may result in a tax liability that must be paid from the trust portfolio rather than by the individual investor. Therefore, we extend our models to cover taxation in the DFTL series. Taxation has a lesser effect on sustainability rates under regime switching models than under models assuming normality in the return distribution.

Figure 4 clearly illustrates that the most simple and commonly used risk model, the NH model, understates risk given any set of underlying inputs or assumptions. On the other hand, assuming markets start out in a bear regime may overstate the risk of retirement failure. Nevertheless, it is always interesting to examine the "worst case" environment. The differences in sustainability rates are stark: a 4% failure rate at the low end versus a 49% failure rate at the high end. Decisions based on implausible risk models may not be appropriate and, thus, may mislead investors in assessing the risks and return expectations of their retirement investment strategies.

6. Utility

Discussions concerning strategies to enhance portfolio sustainability differ from discussions concerning how to optimize aggregate utility of consumption for a retired investor with finite resources. Portfolio sustainability, when defined as the ability to fund a minimum periodic target income, implies a state preference utility function. That is to say, a retired investor may have a strong preference for avoiding periods during which consumption fall below a minimum acceptable threshold. Such an investor is willing to sacrifice greater utility in higher-portfolio-value states in favor of assuring a minimum standard of living in lower-portfolio-value states. Aggregate utility—summed over all consumption/investment states—takes a back seat to assuring, at least, a minimum living standard in each state. Optimization of expected utility, in most retirement income risk models, is a probability-weighted value taken over the entire distribution of outcomes—i.e., over all possible economic states from depression to prosperity. Summing utility values over all states assumes a separable utility function. Furthermore, conclusions derived from optimization procedures may differ drastically from those drawn from sustainability analysis. For example, an optimal withdrawal rate in a study by Finke, Pfau, and Williams (2012) requires that the utility-maximizing investor accept only a 43% sustainability rate. Tomlinson (2012) notices similar observations. Typically, commentators tracking shortfall risk metrics would consider such an optimal withdrawal strategy to be unacceptable.

Fortunately, whenever a threshold level of consumption must be maintained, the two approaches sometimes coincide. The investor may apply a utility penalty—negative utility—for failing to meet threshold income requirements. Some retirement income risk models impose an additional penalty for *exceeding* a periodic income or ending wealth target.[23] The risk model imposes a

---

[23] Hughen, Laatsch, and Klein (2002) view positive terminal wealth as a potential "opportunity cost." The authors develop a concept called "the equivalent payment value." This is a way to convert terminal wealth into extra monthly payments throughout the planning horizon. Blanchett, Kowara, and Chen (2012) optimize based on a

penalty for over-achieving under the supposition that target financial objectives could have been met at a lower level of risk. In terms of consumption objectives, some commentators view surplus ending wealth merely as a missed opportunity to enjoy a higher standard of living throughout retirement.

7. Conclusion

The normal (bell curve) distribution is not a good fit for most financial asset return series. Quantifying investment risk by the first two moments of multivariate symmetric distributions (Gaussian, Student's t, etc.) is often misleading. Furthermore, Monte Carlo simulations based on a normal distribution cannot realistically capture asymmetry in the distribution (skew) or the frequency and magnitude of tail-risk events (leptokurtosis). Risk models inputting return distributions with differing assumptions, operating under different stochastic processes, often produce significantly different results. To mitigate deficiencies, we employ a hybrid autoregressive, regime-switching risk model using two state-dependent normal distributions with separately calculated means and variances. The distributions, according to the Markov transition probabilities, capture the frequency and magnitudes of outlier results better than distributions produced by single-parameter input variables. Given a large number of simulation paths, our retirement income risk model provides a rich set of future asset returns. Although no investment risk model can predict the future, one hallmark of a credible model is that it enables investors to make good decisions within a wide range of possible futures.[24] Success or failure should never

utility function that penalizes portfolio depletion more heavily than it penalizes excess bequests. See also, Scott, Sharpe, and Watson (2009), which advances the proposition that investors should not try to generate "unneeded" / low-utility surplus.

[24] Thomas J. Sargent's Noble Prize winning research deals with how investors make decisions when they doubt the accuracy of their model. When confronted with ambiguity, they tend to use a family of models and to over-weight bad outcomes as a mechanism for exercising caution: see Hansen & Sargent (2008) for example. For a discussion on the wisdom of using multiple models for security valuation, see Collins (2007).

be evaluated in terms of just a single model—nor in terms of just a single metric. A model, in many respects, is just one person's—i.e., the model builder's—opinion about how the future may unfold.

Given the computer power currently available to investment advisors, and given the approximately thirty years of research into econometric topics such as time series analysis, what accounts for the propensity to use oversimplified risk models? Perhaps the financial advice profession suffers from what Paul Kleindorfer (2010) terms 'Legitimation.' He defines the term as follows: "…a credible anticipation of being held accountable not just for outcomes but for the logic that led to them will have predictable effects on the nature of the choice process itself." A disgruntled investor's demands for explanations regarding why and how an advisor's recommendation went sour may lead advisors to "play it safe." In the context of this article, advisors may tend to use only models commonly employed throughout the profession—e.g., historical back testing or Monte Carlo simulation models—so that if the investment strategy fails to produce its intended result, the advisor can take comfort in the fact that most other advisors also got it wrong: "…the mere thought of making choices of consequence under conditions of ambiguity and ignorance calls out for company." Undoubtedly, there are other explanations for the financial industry's slow adoption of academic advances in risk modeling. However, if the central focus of each investor remains the intelligent consideration of risk / return tradeoffs, then the tools of the advisory profession should be evaluated in terms of their ability to indicate the consequences of portfolio management elections.

A model is an imperfect representation of a more complex reality. In this case, there are (at least) two embedded "model risks" to consider:

1. Investors are interested in forecasts of a price change process. However, the time series of asset prices is not statistically stationary (i.e., exhibits the potential for infinite variance). It is only by differentiating the logarithm of prices on a period-by-period basis that the creation of a stationary series of returns is possible. That is to say, it is only possible to model asset *returns*; but an investor measures wealth based on asset *prices*. This is a subtle but important distinction. Returns are based on the single historical path of price changes, the realization of which is merely a manifestation of an unknown price generating process. Simulation analysis greatly broadens our perspective about possible future outcomes; but any model of such a process must remain only a crude approximation of reality. Indeed, calculation of investment return is a function of the measurement interval (yearly, monthly, daily, intraday, continuous time) and, at the limit, may be meaningless in a statistical context.
2. The single historically realized return path for each asset class may be "representative" of the unknown price generating process; or, may merely be an outlier result unlikely to persist. For example, an asset allocation tilt towards small and value stocks is justified based on historical return data. If the premium for investing in small and value stocks reflects a reward for systematic risks, then investors have some confidence that they will continue to be rewarded for making these investments. If, however, the premium for such investments is merely an artifact of a chance historical price change process, then investors may be increasing risk as they move their asset allocation deeper into the small/value gradient. Furthermore, investment volatility is measured by the variance statistic—the squared difference between actual returns and average return. But if the

historical return path is not representative, then the concept of average becomes meaningless and statistical measures are not illuminating.

Beyond this, a savvy investor should be aware of the limitations of basing decisions on the outcome of a risk model. If optimal decisions are model-dependent, how does an investor make the best decision when the outputs of various models differ significantly? This means that the investor must consider both the credibility of each risk model as well as the economic consequences of the various choices that the risk models present. Investors are rewarded for taking prudent and calculated risks. Investors may use historical data to make inferences concerning the interrelationships between asset allocation, risk and reward. However, in designing and implementing a portfolio, it is always wise to remain aware of uncertainties in both data and the risk models that incorporate it. This is why it is important to track both sustainability and feasibility as part of a prudent assessment of retirement income strategies. Past performance is not a guarantee of future results.

BIBLIOGRAPHY

Alexander, C. (2008). *Market Risk Analysis.* John Wiley & Sons.

Ameriks, J., Caplin, A., & Van Nieuwerburgh, S. (2008). Annuity Valuation, Long-Term Care, and Bequest Motives. In J. Ameriks, & O. Mitchell (Eds.), *Recalibrating Retirement Spending and Saving* (pp. 251-275). Oxford University Press.

Ang, A., & Bekaert, G. (2004). How Regimes Affect Asset Allocation. *Financial Analyst Journal, 60*(2), 86-99.

Athavale, M., & Goebel, J. M. (2011). A Safe Safe Withdrawal Rate Using Various Return Distribuitons. *Journal of Financial Planning, 24*(7), 36-43.

Bierwirth, L. (1994). Investing for retirement: using the past to model the future. *Journal of Financial Planning, 7*(1), 14-24.

Blanchett, D. M., & Blanchett, B. C. (2008). Data Dependence and Sustainable Real Withdrawal Rates. *Journal of Financial Planning, 21*(9), 70-84.

Blanchett, D., Kowara, M., & Chen, P. (2012). Optimal Withdrawal Strategy for Retirement Income Portfolios. *Morningstar*. Retrieved from http://corporate.morningstar.com/US/documents/ResearchPapers/OptimalWithdrawalStrategyRetirementIncomePortfolios.pdf

Brown, R. L., & McDaid, J. (2003). Factors Affecting Retirement Mortality. *North American Actuarial Journal, 7*(2), 24-43.

Campbell, J. Y., & Viceira, L. M. (2005). Long-Horizon Mean-Variance Analysis: A User Guide, Appendix for "The Term Structure of the Risk-Return Tradeoff". Financial Analysis Journal. Retrieved from http://scholar.harvard.edu/files/campbell/files/long-horizon_appendix_2005.pdf

Campbell, J. Y., & Viceira, L. M. (2005). The Term Structure of the Risk-Return Trade-Off. *Financial Analysts Journal, 61*(1), 34-44.

Collins, P. J. (2007). Prudence. *The Banking Law Journal*, 3-70.

Collins, P. J., Savage, S. L., & Stampfli, J. (2000). Financial Consequences of Distribution Elections from Total Return Trusts. *Real Property, Probate, and Trust Journal, 35*(2), 243-3004.

Cooley, P. L., Hubbard, C. M., & Walz, D. T. (2011). Portfolio Success Rates: Where to Draw the Line. *Journal of Financial Planning*, 48-60.

Coopersmith, L. W., & Sumutka, A. R. (2011). Tax Efficient Retirement Withdrawal Planning Using a Linear Programming Model. *Journal of Financial Planning, 24*(9), 50-59.

Davis, J. H., Wicas, N. W., & Kinniry, F. M. (2004). The Strengths and Weaknesses of Various Financial Simulation Methods. *Journal of Wealth Management, 6*(4), 33-42.

DiCarlo Jr., D. P., & Fast, S. M. (2008). Prudence: What Are the Odds? *ALI-ABA Course of Study: Representing Estate and Trust Beneficiaries and Fiduciaries*, 477-479.

Finke, M., Pfau, W. D., & Williams, D. (2012). Spending Flexibility and Safe Withdrawal Rates. *Journal of Financial Planning, 25*(3), 44-51.

Gupta, A., & Li, Z. (2013). Optimal Annuity Purchase Decisions Under Uncertain Lifetime. *Journal of Family Economics, 34*, 447-459.

Hamilton, J., & Susmel, R. (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics, 64*, 307-333.

Hansen, L. P., & Sargent, T. J. (2008). *Robustness.* Princeton University Press.

Hardy, M. (2003). *Investment Guarantees: Modeling and Risk Management for Equity-Linked LIfe Insurance.* John Wiley & Sons.

Huang, H., Milevsky, M. A., & Wang, J. (2004). Ruined Moments in Your Life: How Good are the Approximations? *Insurance: Mathematics and Economics, 34*(3), 421-447.

Hughen, C. J., Laatsch, F. E., & Klein, D. P. (2002). Withdrawal Patterns and Rebalancing Costs for Taxable Portfolios. *Financial Services Review, 11*, 341-366.

Kaplan, P. D. (2006). Asset Allocation with Annuities for Retirement Income Management. *THe Journal of Wealth Management, 8*(4), 27-40.

Karolyi, A. G. (2001). Why Stock Return Volatility Really Matters. *Strategic Investor Relations*.

Kleindorfer, P. R. (2010). Reflections on Decision-making under Uncertainty. In F. X. Diebold, N. A. Doherty, & R. J. Herring (Eds.), *The Known, the Unknown, and the Unknowable in Financial Risk Management.* Princeton University Press.

Klinger, W. (2011). In Search of the "Best" Retirement Strategy. *Journal of Financial Service Professionals, 65*(1), 62-73.

Kopcke, R. W., Webb, A., Hurwitz, J., & Li, Z. (2013). Rethinking Optimal Wealth Accumulation and Decumulation Strategies in the Wake of the Financial Crisis. *Center for Retirement Research at Boston College CRR WP*.

Kritzman, M., Page, S., & Turkington, D. (2012). Regime Shifts: Implications for Dynamic Strategies. *Financial Analysts Journal, 68*(3), 22-39.

Laverty, W. H., Miket, M. J., & Kelly, I. W. (2002). Simulation of Hidden Markov Models with Excel. *Journal of the Royal Statistical Society: Series D (The Statistician), 51*(1), 31-40.

Lee, M. I. (2009). Model Risk in Retirement Simulations. Retrieved from http://us.dimensional.com/pdf/model_risk_in_retirement_simulations.pdf

Levy, H., & Duchin, R. (2004). Asset Return Distribuitons and the Investment Horizon. *Journal of Portfolio Management, 30*(3), 47-62.

Litzenberger, R. H., & Modest, D. M. (2010). Crisis and Noncrisis Risk in Financial Markets. In F. X. Diebold, N. A. Doherty, & R. J. Herring (Eds.), *The Known, the Unknown and the Unknowable in Financial Risk Management* (pp. 74-102). Princeton University Press.

McGoun, E. G. (1995). The History Of Risk "Measurement". *Critical Perspectives on Accounting, 6*(6), 511-532.

Milevsky, M. A., & Abaimova, A. (2006). Will the True Monte Carlo Number Please Stand Up? *Journal of Financial Planning*, online edition.

O'Flinn, C., & Schirripa, F. (2010). Retrieved from Social Science Research Network: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1641382

Orszag, M. (2002). *Ruin in Retirement: Running Out of Money in Drawdown Programs.* Watson Wyatt Worldwide Technical Report RU06.

Pang, G., & Warshawsky, M. J. (2009). Comparing Strategies for Retirement Wealth Management: Mutal Funds and Annuities. *Journal of Financial Planning, 22*(8), 36-47.

Pye, G. B. (2001). Adjusting Withdrawal Rates For Taxes And Expenses. *Journal of Financial Planning, 14*(4), 126-136.

Scott, J., Sharpe, W., & Watson, J. (2009). The 4% Rule - At What Price? *Journal of Investment Management*.

Smith, G., & Gould, D. (2006). Measuring and Controlling Shortfall Risk in Retirement. *The Journal of Investing, 16*(1), 82-95.

Spitzer, J. J. (2008). Retirement withdrwals: an analysis of the benefits of periodic "midcourse" adjustments. *Financial Services Review, 17*(1), 17-29.

Tomlinson, J. A. (2012). A Utility-Based Approach to Evaluating Investment Strategies. *Journal of Financial Planning, 25*(2), 53-60.

Turner, J. A., & Witte, H. A. (2009). Retirement Planning Software and Post-Retirement Risks. *Society of Actuaries*, p. 20.