

# **$R^2$ and the benefits of multiple-fund portfolios**

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Comments are enormously welcome!

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## ABSTRACT

Recent research shows superior performance of equity funds which differ the most from their benchmarks. However, existing studies have not examined important questions about risk, such as the impact of holding a portfolio of low  $R^2$  funds on the reduction in portfolio risk achieved through diversifying across funds. Our study provides some answers. Using a variety of risk metrics, we evaluate the benefits of diversification among both monthly and annually reconstituted portfolios, where funds are sorted into portfolios based on their quintile rank of  $R^2$ . We find that the benefits of diversification are most pronounced within a portfolio of low  $R^2$  funds and that the benefits are increasing in the length of the reconstitution period.

## 1. Introduction

Several recent studies demonstrate that the more different a mutual fund is from its benchmark index, the stronger its performance. One approach to determining a fund's uniqueness is to calculate its Active Share, which equals the percentage of its holdings that do not overlap with those in its benchmark index. Active Share was developed by Cremers and Petajisto (2009), who show that funds ranked in the top quintile of Active Share outperform their benchmarks by more than 1% per year after fees and transactions costs.

An alternative approach to measuring uniqueness is presented by Amihud and Goyenko (2013). They focus on  $R^2$  values derived through the use of Carhart's (1997) four-factor model and Cremers, Petajisto, and Zitzewitz's (2013) alternative index-based four-factor model. Regressing an index fund on its benchmark should produce an  $R^2$  that is very close to 1.00. A lower  $R^2$  value implies that a fund has deviated from its benchmark because its management is selective in allocating capital amongst securities. Amihud and Goyenko show that performance is monotonically decreasing in  $R^2$ . For example, funds ranked in the bottom quintile of  $R^2$  produce annual alpha of 0.6% while those ranked in the top quintile generated annual alpha of -1.5%.

The  $R^2$  measure employed by Amihud and Goyenko (2013) is particularly appealing to practitioners because it requires only return data and a computer program that can run an ordinary least squares regression.

A basic tenet of Modern Portfolio Theory is that the idiosyncratic risk of a portfolio of individual securities can be eliminated through diversification. This is one of the main reasons why households choose to hold mutual funds (Gruber, 1996). However, there are considerable differences in  $R^2$  values across funds, which motivates the purpose of this paper: to investigate

the advantages of diversifying among funds when employing Amihud and Goyenko's (2013) strategy of investing in low  $R^2$  funds.

To investigate the benefits of diversification among low  $R^2$  funds, we sort funds into portfolios based on their quintile rank of  $R^2$ . Using a variety of risk metrics, we evaluate the benefits among both monthly and annually reconstituted portfolios. We find that the benefits are most pronounced within a portfolio of low  $R^2$  funds and that the benefits are increasing in the length of the reconstitution period.

## 2. Data and Methodology

To evaluate the potential benefits of diversifying among low  $R^2$  funds, we obtain monthly net-of-expense returns and total net assets (TNA) from Morningstar Direct's survivor-bias-free United States Mutual Funds database<sup>1</sup> on all open-end equity funds which have a U.S. broad asset class of "U.S. Stock."<sup>2</sup> Monthly returns on share classes are aggregated to the portfolio level by weighting them by their TNA as of the end of the previous month. The time period of the study was December 1995 through March 2013 and the data were collected on May 13, 2013. December 1995 was chosen as the initial month of the study because the number of distinct mutual fund portfolios with valid return and TNA data increased to 1,000 in that month.

We estimate rolling  $R^2$ s for each actively managed mutual fund, which we define as funds that are not classified by Morningstar as either an "index fund" or an "enhanced index fund".  $R^2$ s are estimated over the prior 24 months through the use of Carhart's (1997) four-factor

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<sup>1</sup> Morningstar Direct is the most complete and timely database offered by Morningstar, Inc., a leading provider of mutual fund data.

<sup>2</sup> Other asset classes are Balanced, Commodities, International Stock, Money Market, Municipal Bond, Sector Stock, and Taxable Bond. Morningstar does not assign funds to multiple asset classes.

model and Cremers, Petajisto, and Zitzewitz's (2013) alternative index-based four-factor model. The specification of the standard Carhart (1997) model is

$$r_{i,t} = \alpha_i + \beta_{m,i}r_{m,t} + \beta_{smb,i}r_{smb,t} + \beta_{hml,i}r_{hml,t} + \beta_{mom,i}r_{mom,t} + \varepsilon_{i,t}, \quad (1)$$

where  $r_{i,t}$  is fund  $i$ 's net-of-expense return in month  $t$  in excess of the risk-free rate of interest, which is proxied by the interest rate on one-month Treasury bills (from Ibbotson Associates).  $r_{m,t}$  is the value-weighted return on all stocks traded on the NYSE or NASDAQ in excess of the risk-free rate.  $r_{smb,t}$  and  $r_{hml,t}$  are the returns on Fama and French's (1993) portfolios that proxy for the common risk factors of size and book-to-market.  $r_{mom,t}$  is the return on Carhart's (1997) portfolio that proxies for momentum in stock returns. The returns associated with the research factors are gathered from Kenneth French's website.<sup>3</sup>  $\alpha_i$  captures fund  $i$ 's net-of-expense performance that cannot be explained by the factors in the model.

Cremers, Petajisto, and Zitzewitz (2013) show that the Carhart (1997) model produces biased estimates of alpha due to flaws in the construction of the factors used in the model. To overcome these shortcomings, they developed an alternative benchmark model that uses orthogonalized returns on major benchmark indices to mimic the size and value factors in stock returns. The specification of Cremers, Petajisto, and Zitzewitz's alternative index-based four-factor model is

$$r_{i,t} = \alpha_i + \beta_{s5,i}r_{s5,t} + \beta_{r2-s5,i}r_{r2-s5,t} + \beta_{r3v-r3g,i}r_{r3v-r3g,t} + \beta_{mom,i}r_{mom,t} + \varepsilon_{i,t}, \quad (2)$$

where  $r_{s5,t}$  is the return on the S&P 500 index in excess of the risk-free rate,  $r_{r2-s5,t}$  is the return on the Russell 2000 index minus the return on the S&P 500, and  $r_{r3v-r3g,t}$  is the return on the Russell 3000 Value index minus the return on the Russell 3000 Growth index.  $r_{s5,t}$  represents the

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<sup>3</sup> Details on the construction of the variables gathered from Kenneth French's website can be found at [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data\\_Library/f-f\\_factors.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html) and [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data\\_Library/det\\_mom\\_factor.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_mom_factor.html).

market risk factor in stock returns while  $r_{r2-s5,t}$  represents the size factor and  $r_{r3v-r3g,t}$  represents the value factor. The returns on the benchmark indices are gathered from Morningstar Direct.

Next, we conduct simulations to compute returns on portfolios that invest in randomly selected funds that rank in a given quintile of  $R^2$ . The number of funds,  $n$ , held in these portfolios ranges from 1 to 200. For each value of  $n$ , 1000 simulations are performed. At the beginning of each month, starting in December 1997, capital is equally allocated across  $n$  randomly selected funds that rank in a given quintile of  $R^2$  based on current information on  $R^2$ s. The performance of an alternative strategy involving annual reconstitution is also evaluated.<sup>4</sup> The graphics in Figure 1 show that there is a modest amount of variation in the quintile rank of  $R^2$ s among mutual funds over time. Therefore, the benefit of the first strategy is that it minimizes drift in a portfolio's fund-level  $R^2$  exposure. The benefit of the second strategy is that it is more practical to implement due to the presence of taxes and transaction costs.

### 3. Results – Monthly Reconstituted Portfolios

Table I displays the average  $R^2$ , standard deviation, idiosyncratic volatility, and market beta of the constituents of the  $R^2$ -sorted portfolios examined in this study<sup>5</sup>. There is little difference in the average standard deviations or betas across the  $R^2$ -sorted portfolios yet the idiosyncratic volatilities monotonically decrease by 5.92 to 6.45 percentage points (40.9% to 43.4%) across the portfolios depending on weighting and asset pricing model specification.

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<sup>4</sup> In the event that a fund dies it is replaced with another fund in the same quintile-rank of  $R^2$  based on updated information on  $R^2$ s.

<sup>5</sup> We define idiosyncratic volatility as the standard deviation of the error term from the employed regression model.

### 3.1 Standard Deviation

Following the approach used by Evans and Archer (1968) for individual stocks, Figure 2 conveys the relationship between the number of funds held in a portfolio and the average annualized time-series standard deviation of portfolio returns across the 1000 iterations for that portfolio size. Each graphic shows the number of funds,  $n$ , ranging from 1 to 200. To facilitate interpretation, these graphics are accompanied by tabular output in Table 2. Table 2 also presents the results for passively managed funds<sup>6</sup>.

Consistent with our hypothesis, the benefits of diversification are more pronounced for portfolios of low  $R^2$  funds. Graphic A conveys the results derived through the use of Carhart's (1997) four-factor model. The average standard deviation of a portfolio that invests in funds that rank in the bottom quintile of  $R^2$  is 20.67% for  $n = 1$ , 17.52% for  $n = 10$ , 17.21% for  $n = 50$ , and 17.19% for  $n = 200$ . These correspond to a 15.2%, 16.8%, and 16.8% reduction in volatility when  $n$  is expanded from 1 to 10, 50, and 200 respectively. In contrast, the volatility of an "All Active Funds" portfolio, which randomly allocates capital across the active funds regardless of  $R^2$ , is reduced by 11.0%, 12.0%, and 12.3% when  $n$  is expanded from 1 to 10, 50, and 200 respectively. Graphic B compares the bottom quintile portfolio to the other four portfolios that are formed based on quintile rank of  $R^2$ . The graphic further illustrates that the benefits of diversification are decreasing in  $R^2$ . Graphics C and D convey the results derived through the use of Cremers, Petajisto, and Zitzewitz's (2013) alternative index-based four-factor model. The results are robust to those derived through the standard Carhart four-factor model. For example,

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<sup>6</sup> We define passively managed funds as those that are classified by Morningstar as an "Index Fund" but not an "Enhanced Index Fund".

the volatility of the bottom quintile portfolio is reduced by 15.3%, 16.8%, and 17.1% when  $n$  is expanded from 1 to 10, 50, and 200 respectively.

From analyzing the relationships between standard deviation and  $n$ , it is clear that the percentage decrease in volatility from expanding  $n$  is greater for a portfolio of low  $R^2$  funds than for the general universe of actively managed funds. Additionally, 90% of the decrease in volatility associated with expanding  $n$  from 1 to 200 is achieved with expanding  $n$  to around 10. These results are reminiscent of Evans and Archer's (1968) finding that "much of the unsystematic variation is eliminated by the time the 8th security is added to the portfolio." That finding led to the standard textbook advice about eight to 20 stocks being enough for adequate diversification in a portfolio of individual stocks. However, in recent years a variety of different research methodologies suggest that far more than 20 stocks are needed (Domian, Louton, and Racine (2003, 2007) and Statman (2004). In the remainder of our study we use a shortfall risk approach to explore the benefits of diversification in a portfolio of low  $R^2$  funds.

### *3.2 Return Distributions*

While the time-series standard deviation of returns is a commonly used measure of risk, Utility Theory suggests that investors are most concerned about shortfall risk, the possibility of the terminal value of a portfolio not achieving a target. Additionally, in a study using stock returns, Domian, Louton, and Racine (2007) show that due to nonnormality in the distribution of asset returns, standard deviation is a rather weak proxy for shortfall risk. Therefore, we analyze the probability distributions of average returns and terminal wealth for portfolios of 1 to 200 funds.



Table 3 conveys the average annualized monthly returns on portfolios formed based on quintile rank of  $R^2$  derived through the use of Carhart's (1997) four-factor model. Unsurprisingly, in the aggregate, actively managed funds fail to outperform passively managed funds. For example, when examining the median returns when  $n = 50$ , the "All Active Funds" portfolio underperformed the "All Passive Funds" portfolio by 0.02 percentage points per year. However, consistent with Amihud and Goyenko (2003), the results wildly change when they are conditioned upon  $R^2$ . For example, when examining the median returns when  $n = 50$ , the "Quintile 1" portfolio of actively managed funds outperformed the "All Passive Funds" portfolio by 2.52 percentage points per year.

When evaluated at the 5<sup>th</sup> percentile of the distribution, the returns on the portfolio that invests in bottom quintile funds are 4.58% for  $n = 1$ , 8.32% for  $n = 10$ , 9.26% for  $n = 50$ , and 9.80% for  $n = 200$ . These values correspond to a 3.74, 4.68, and 5.22 percentage point improvement in the 5<sup>th</sup> percentile value of returns when  $n$  is expanded from 1 to 10, 50, and 200 respectively. In contrast, the improvement in the 5<sup>th</sup> percentile values of returns on the "All Active Funds" portfolio is 2.58, 3.30, and 3.61 percentage points when  $n$  is expanded from 1 to 10, 50, and 200 respectively. 72% of the 5.22 percentage point improvement in the 5<sup>th</sup> percentile value of returns on the portfolio of bottom quintile funds when  $n$  is expanded from 1 to 200 is achieved with a more modest expansion of  $n$  from 1 to 10. 90% of the improvement is achieved with an expansion of  $n$  from 1 to 50.

When evaluated at the stricter 1<sup>st</sup> percentile of the distribution, the returns on the portfolio that invests in bottom quintile funds are 2.57% for  $n = 1$ , 7.65% for  $n = 10$ , 8.99% for  $n = 50$ , and 9.73% for  $n = 200$ . These values correspond to a 5.08, 6.43, and 7.16 percentage point improvement in the 1<sup>st</sup> percentile value of returns when  $n$  is expanded from 1 to 10, 50, and 200

respectively. In contrast, the improvement in the 1<sup>st</sup> percentile values of returns on the “All Active Funds” portfolio is 4.30, 5.19, and 5.64 percentage points when  $n$  is expanded from 1 to 10, 50, and 200 respectively. 71% of the 7.16 percentage point improvement in the 1<sup>st</sup> percentile value of returns on the portfolio of bottom quintile funds when  $n$  is expanded from 1 to 200 is achieved with a more modest expansion of  $n$  from 1 to 10. 90% of the improvement is achieved with an expansion of  $n$  from 1 to 50. The results derived through the use of Cremers, Petajisto, and Zitzewitz’s (2013) alternative index-based four-factor model, conveyed in Table 4, are consistent with those derived through the traditional Carhart (1997) four factor model.

Taken together, our analysis of the return distributions are similar to that of the standard deviations in that the improvement in 1<sup>st</sup> and 5<sup>th</sup> percentile values of return from expanding  $n$  is greater for a portfolio of low  $R^2$  funds than for the general universe of actively managed funds. However, when the benefit of diversification is gauged by the improvement in 1<sup>st</sup> and 5<sup>th</sup> percentile values of return rather than standard deviation, many more funds are needed to achieve the benefits of being diversified across 200 funds. For example, it takes roughly 50 funds to obtain 90% of the improvement in 1<sup>st</sup> and 5<sup>th</sup> percentile values of return associated with expanding  $n$  from 1 to 200. This corroborates Domian, Louton, and Racine’s (2007) analysis of the benefits of diversification among individual stocks.

It is also interesting to note that when held in a diversified portfolio, low  $R^2$  funds can be used to mitigate shortfall risk. For example, when  $n \geq 10$ , the 1<sup>st</sup> percentile values of return on the portfolios that invests in bottom quintile funds consistently beat the median values of return on not only the “All Active Funds” but also the “All Passive Funds” portfolios. Furthermore, when  $n \geq 50$ , the 1<sup>st</sup> percentile values of return on the portfolios that invests in bottom quintile

funds consistently beat the 99<sup>th</sup> percentile values of return on both the “All Active Funds” and the “All Passive Funds” portfolios.

### *3.3 Terminal Wealth Distributions*

The implications of the analysis of terminal wealth distributions are similar to those obtained through the analysis of the return distributions. Table 5 conveys the terminal wealth for portfolios formed based on quintile rank of  $R^2$  derived through the use of Carhart’s (1997) four-factor model. When evaluated at the 5<sup>th</sup> percentile of the distribution, the wealth accumulated in March 2013 from investing \$1 in a portfolio of funds in the bottom quintile of  $R^2$  in December 1997 is \$1.38 when  $n = 1$ , \$2.68 when  $n = 10$ , \$3.10 when  $n = 50$ , and \$3.28 when  $n = 200$ . These values correspond to a \$1.30, \$1.72, and \$1.90 decrease in Jorion’s (2000) Value at Risk measure at the 5% confidence level when  $n$  is expanded from 1 to 10, 50, and 200 respectively. In contrast, the decrease in Value at Risk at the 5% confidence level on the “All Active Funds” portfolio is merely \$0.70, \$0.93, and \$1.03 when  $n$  is expanded from 1 to 10, 50, and 200 respectively. 69% of the \$1.90 drop in Value at Risk at the 5% confidence level on the portfolio of bottom quintile funds when  $n$  is expanded from 1 to 200 is achieved with a more modest expansion of  $n$  from 1 to 10. 90% of the reduction is achieved with an expansion of  $n$  from 1 to 50.

When evaluated at the 1<sup>st</sup> percentile of the distribution, the wealth accumulated in March 2013 from investing \$1 in a portfolio of funds in the bottom quintile of  $R^2$  in December 1997 is \$1.02 when  $n = 1$ , \$2.44 when  $n = 10$ , \$2.97 when  $n = 50$ , and \$3.25 when  $n = 200$ . These values correspond to a \$1.42, \$1.95, and \$2.23 decrease in Value at Risk at the 1% confidence level when  $n$  is expanded from 1 to 10, 50, and 200 respectively. In contrast, the decrease in

Value at Risk at the 1% confidence level on the “All Active Funds” portfolio is merely \$0.93, \$1.19, and \$1.33 when  $n$  is expanded from 1 to 10, 50, and 200 respectively. 64% of the \$2.23 drop in Value at Risk at the 1% confidence level on the portfolio of bottom quintile funds when  $n$  is expanded from 1 to 200 is achieved with a more modest expansion of  $n$  from 1 to 10. 87% of the reduction is achieved with an expansion of  $n$  from 1 to 50. The results derived through the use of Cremers, Petajisto, and Zitzewitz’s (2013) alternative index-based four-factor model, displayed in Table 6, are consistent with those derived through the traditional Carhart (1997) four factor model.

The implications of the analysis of terminal wealth distributions are similar to those obtained through the analysis of the return distributions. First, the decrease in Value at Risk at the 1% and 5% confidence level from expanding  $n$  is larger for a portfolio of low  $R^2$  funds than for the general universe of actively managed funds. Second, when risk is gauged by Value at Risk rather than by standard deviation, the benefits of diversification extend far beyond  $n = 10$ . Third, within a diversified portfolio, low  $R^2$  funds can mitigate shortfall risk. For example, when  $n \geq 10$ , the 1<sup>st</sup> percentile values of ending wealth on the portfolio that invests in bottom quintile funds consistently beat the median values on both the “All Active Funds” and the “All Passive Funds” portfolios. When  $n \geq 50$  they consistently beat the 99<sup>th</sup> percentile values of return on both of the aggregate portfolios.

#### **4. Results – Annually Reconstituted Portfolios**

In this section we explore how the results change when portfolios are reconstituted once a year instead of once a month. Due to limited availability of TNA data in the earlier months of this study, we use equal- rather than TNA-weighted returns on the mutual funds. Many investors (and

their financial advisors) might reject the monthly reconstitution approach as being too time-consuming and costly due to taxes and trading costs. Our presentation parallels the findings from the earlier sections of the paper.

#### *4.1 Standard Deviation*

Figure 3 and Table 7 present the average annualized standard deviations for portfolios involving the use of the Carhart model that are reconstituted annually. These results continue to show more pronounced diversification benefits for the Quintile 1 funds. For the Quintile 1 funds, the average standard deviation is 20.91% for  $n = 1$ , 17.96% for  $n = 10$ , 17.63% for  $n = 50$ , and 17.57% for  $n = 200$ . These correspond to a 14.1%, 15.7%, and 16.0% reduction in volatility when  $n$  is expanded from 1 to 10, 50, and 200 respectively. In contrast, the volatility of the “All Active Funds” portfolio is reduced by 10.5%, 11.6%, and 11.8% when  $n$  is expanded from 1 to 10, 50, and 200 respectively. Furthermore, in Graphic B and Table 7 we see that as before, the benefits of diversification are decreasing in  $R^2$ . Graphics C and D, and Panel B in Table 7, show similar findings from the Cremers, Petajisto, and Zitzewitz model.

#### *4.2 Return Distributions*

Table 8 presents average annualized returns in the same format as Table 3. Both tables form quintiles based on  $R^2$  values from the Carhart model. The only difference is in the frequency of reconstitution. In comparison to the earlier results, the annual reconstitution in Table 8 produces modestly lower returns on the Quintile 1 funds, and higher ranges across the percentiles.

When evaluated at the 5<sup>th</sup> percentile of the distribution, the returns on portfolios that invest in bottom quintile funds are 3.00% for  $n = 1$ , 7.38% for  $n = 10$ , 8.53% for  $n = 50$ , and 9.14% for  $n = 200$ . In comparison, the 5<sup>th</sup> percentile value of returns on the monthly reconstituted portfolios were 4.58% for  $n = 1$ , 8.32% for  $n = 10$ , 9.26% for  $n = 50$ , and 9.80% for  $n = 200$ . In analyzing the differences between reconstitution frequencies, there are two main observations. First, when evaluated at the 5<sup>th</sup> percentile of the distribution, monthly reconstitution produces a slightly higher rate of return than annual reconstitution, particularly for smaller  $n$  portfolios. Second, the benefit of diversification is most pronounced when annual reconstitution is exercised.

When evaluated at the stricter 1<sup>st</sup> percentile of the distribution, the returns on portfolios that invest in bottom quintile funds are -1.32% for  $n = 1$ , 6.55% for  $n = 10$ , 8.27% for  $n = 50$ , and 9.02% for  $n = 200$ . In comparison, the 1<sup>st</sup> percentile value of returns on the monthly reconstituted portfolios were 2.57% for  $n = 1$ , 7.65% for  $n = 10$ , 8.99% for  $n = 50$ , and 9.73% for  $n = 200$ . In comparison to the results from the more relaxed 5<sup>th</sup> percentile of the return distribution, these results show greater dominance of the monthly reconstitution strategy over the annual reconstitution strategy and greater gains to diversification when annual reconstitution is employed. Table 9 shows that the findings are largely unaffected when the quintiles are formed using  $R^2$  values from the Cremers, Petajisto, and Zitzewitz model, instead of the Carhart model.

#### *4.3 Terminal Wealth Distributions*

Table 10 presents terminal wealth distributions involving the use of the Carhart model in the same format as Table 5. When evaluated at the 5<sup>th</sup> percentile of the distribution, the wealth accumulated in March 2013 from investing \$1 in a portfolio of funds in the bottom quintile of  $R^2$

in December 1997 is \$1.11 for  $n = 1$ , \$2.31 for  $n = 10$ , \$2.76 for  $n = 50$ , and \$3.01 for  $n = 200$ . In comparison, the 5<sup>th</sup> percentile values of terminal wealth on the monthly reconstituted portfolios were \$1.38 when  $n = 1$ , \$2.68 when  $n = 10$ , \$3.10 when  $n = 50$ , and \$3.28 when  $n = 200$ .

When evaluated at the 1<sup>st</sup> percentile of the distribution, the wealth accumulated on portfolios that invest in bottom quintile funds is \$0.47 for  $n = 1$ , \$2.04 for  $n = 10$ , \$2.64 for  $n = 50$ , and \$2.95 for  $n = 200$ . In comparison, the 1<sup>st</sup> percentile values of terminal wealth on the monthly reconstituted portfolios were \$1.02 when  $n = 1$ , \$2.44 when  $n = 10$ , \$2.97 when  $n = 50$ , and \$3.25 when  $n = 200$ . Consistent with the implications from the analysis of return distributions, monthly reconstitution manages risk better than annual reconstitution. Table 11 shows that the findings are largely unaffected when the quintiles are formed using  $R^2$  values from the Cremers, Petajisto, and Zitzewitz model, instead of the Carhart model.

## 5. Conclusion

Recent research demonstrates superior performance among equity mutual funds which differ from their benchmarks. Whether measured directly by fund holdings, or indirectly by  $R^2$  values, the funds which differ the most have the strongest performance. However, the literature has not examined risk management issues such as potential benefits from holding an assortment of low  $R^2$  funds.

In our study, we sort funds into groups based on their quintile rank of  $R^2$  and examine differences in the risk characteristics of portfolios of various sizes and fund-level  $R^2$  profiles. Three main points can be drawn from our empirical analysis.

First, the benefit of diversifying across funds is especially pronounced for portfolios that hold low  $R^2$  funds. Using an approach introduced by Evans and Archer (1968) for individual stocks, we show that diversifying across funds results in greater reductions to the standard deviation of a portfolio that invests in funds that rank in the lowest quintile of  $R^2$  than its counterparts. Moreover, diversification also results in greater reductions to shortfall risk at the 1<sup>st</sup> and 5<sup>th</sup> percentiles within a portfolio of funds that rank in the lowest quintile of  $R^2$  than its counterparts.

Second, we find that diversification is even more important if portfolios of low  $R^2$  funds are reconstituted once a year, instead of once a month. When an annual reconstitution strategy is employed, there is a greater improvement in the 1<sup>st</sup> and 5<sup>th</sup> percentile values of the distribution of returns from an expansion in the number of funds held. As before, the diversification benefits are greater for a portfolio of funds that rank in the lowest quintile of  $R^2$  than for its counterparts.

Third, when held in a diversified portfolio, low  $R^2$  funds can be used to mitigate shortfall risk. For example, when the number of funds,  $n$ ,  $\geq 10$ , the 1<sup>st</sup> percentile values of return on the portfolio that invests in bottom quintile funds consistently beat the median values of return on not only an “All Active Funds” but also a “All Passive Funds” portfolio. Furthermore, when  $n \geq 50$ , the 1<sup>st</sup> percentile values of return on the portfolio that invests in bottom quintile funds consistently beat the 99<sup>th</sup> percentile values of return on both the “All Active Funds” and the “All Passive Funds” portfolios.

Our findings suggest there are valuable opportunities for individual investors and financial planners to pursue low  $R^2$  investment strategies. Annual reconstitution may be acceptable as a practical approach, and it showed a very high probability of outperforming index funds during our sample period.



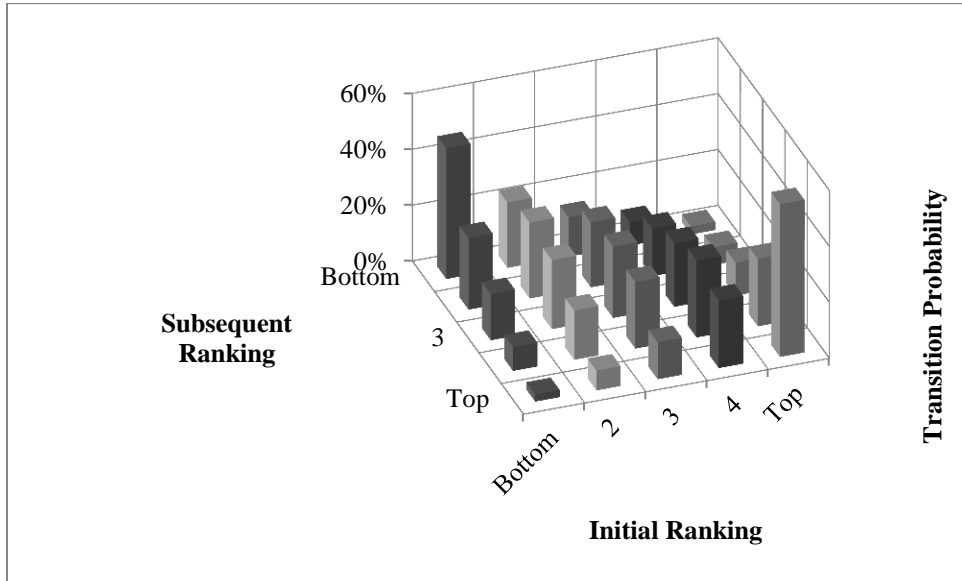
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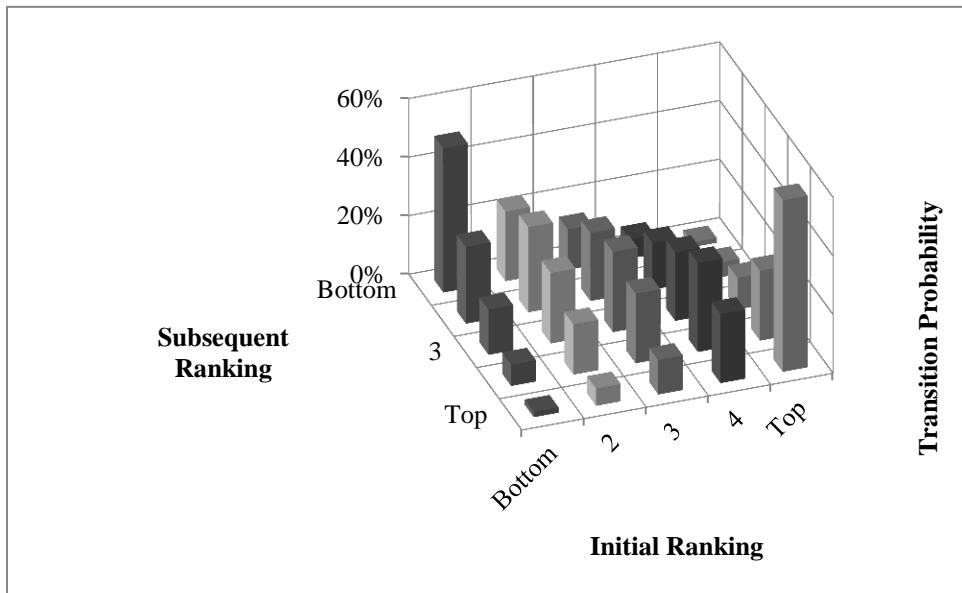
**Figure 1**  
**Contingency Tables of  $R^2$  Rankings**

The bars in graphics A and B indicate the percentage of U.S. Stock mutual funds ranked in quintile  $i$  that are ranked in quintile  $j$  24 months later based on  $R^2$ s derived over a 24 month estimation period. Table A involves the use of the Carhart (1997) four-factor model. Table B involves the use of Cremers, Petajisto, and Zitzewitz's (2013) alternative index-based model. The excess returns on the stock market from December 1995 through March 2013 are from Kenneth French's website. The returns on the research factors and the risk-free rate are from Kenneth French's website. The returns on mutual funds and the indices used in Cremers, Petajisto, and Zitzewitz's model are from Morningstar Direct.

Graphic A: Carhart Model



Graphic B: Cremers, Petajisto, and Zitzewitz Model



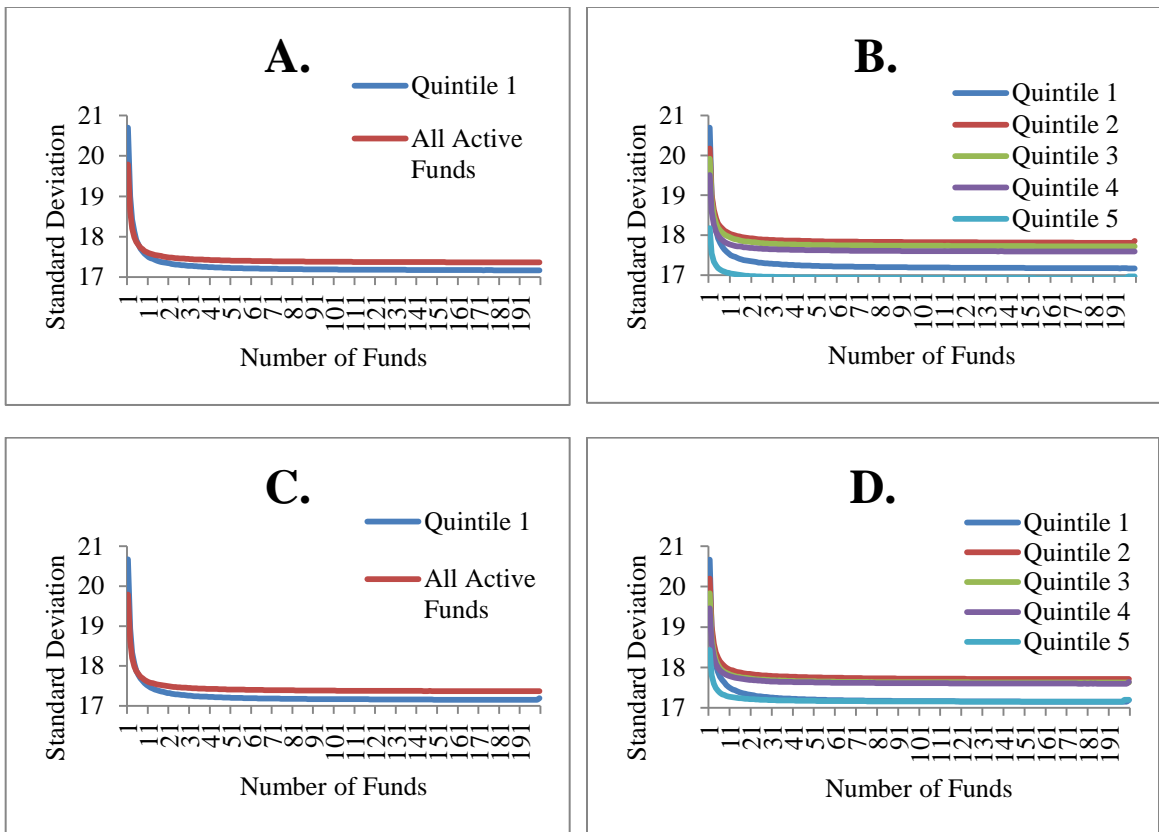
**Table 1****Characteristics of Portfolio Constituents by Quintile of  $R^2$** 

This table displays the average  $R^2$ , standard deviation, idiosyncratic volatility, and beta of funds that are held in the portfolios of U.S. Stock mutual funds that are reconstituted monthly based on quintile rank of trailing  $R^2$ . All characteristics are derived over a 24 month estimation period. The values of these characteristics are reported as time-series means of the cross-sectional averages. The holding period is from December 1997 through March 2013. Standard deviations are annualized through multiplying monthly values by the square root of 12. Panels A and B present the results involving the use of the Carhart (1997) four-factor model to estimate  $R^2$ . Panels C and D use Cremers, Petajisto, and Zitzewitz's (2013) alternative index-based model to estimate  $R^2$ . Panels A and C weight the characteristics equally across funds. Panels B and D weight the characteristics by TNA across the funds. The returns on the research factors and the risk-free rate are from Kenneth French's website. The returns on mutual funds and the indices used in Cremers, Petajisto, and Zitzewitz's model are from Morningstar Direct.

<i>Panel A: Equally-Weighted Characteristics of Portfolio Constituents – Carhart Model</i>					
	Low	2	3	4	High
$R^2$	0.82	0.90	0.93	0.95	0.98
Standard deviation	18.34%	18.72%	18.71%	18.53%	17.63%
Idiosyncratic volatility	14.93%	13.00%	11.87%	10.73%	8.80%
Beta	0.97	1.02	1.02	1.02	1.01
<i>Panel B: TNA-Weighted Characteristics of Portfolio Constituents – Carhart Model</i>					
	Low	2	3	4	High
$R^2$	0.83	0.91	0.93	0.95	0.98
Standard deviation	17.84%	17.81%	17.73%	17.42%	16.97%
Idiosyncratic volatility	14.48%	12.62%	11.52%	10.35%	8.56%
Beta	0.96	1.00	1.00	0.99	1.00
<i>Panel C: Equally-Weighted Characteristics of Portfolio Constituents – Cremers, Petajisto, and Zitzewitz Model</i>					
	Low	2	3	4	High
$R^2$	0.82	0.91	0.94	0.96	0.98
Standard deviation	18.38%	18.75%	18.81%	18.58%	17.39%
Idiosyncratic volatility	14.86%	12.83%	11.66%	10.48%	8.41%
Beta	0.94	1.00	1.01	1.02	1.01
<i>Panel D: TNA-Weighted Characteristics of Portfolio Constituents – Cremers, Petajisto, and Zitzewitz Model</i>					
	Low	2	3	4	High
$R^2$	0.83	0.91	0.94	0.96	0.98
Standard deviation	17.85%	18.06%	18.07%	17.61%	16.54%
Idiosyncratic volatility	14.42%	12.52%	11.40%	10.17%	8.17%
Beta	0.95	0.99	1.00	1.00	0.99

**Figure 2**  
**Number of Funds and Standard Deviation**

These graphics display the relationship between the number of funds held in a portfolio and the time-series standard deviation of the returns on the portfolio. The holding period is from December 1997 through March 2013. The standard deviations are averaged across 1000 simulations. The portfolios equally allocate capital to randomly selected funds that rank in a particular quintile of  $R^2$ . The portfolios are reconstituted at the beginning of each month. Graphics A and C plot this relationship for a portfolio of funds ranked in the bottom quintile of  $R^2$  and a “All Active Funds” portfolio, consisting of all of the funds that comprise the  $R^2$ -sorted portfolios. Graphic B and D plot this relationship for funds ranked in each quintile of  $R^2$ . Graphics A and B use the Carhart (1997) four-factor model to estimate  $R^2$ . Graphics C and D use Cremers, Petajisto, and Zitzewitz’s (2013) alternative index-based model to estimate  $R^2$ . The  $R^2$  estimation period is 24 months. The returns on the research factors and the risk-free rate are from Kenneth French’s website. The returns on mutual funds and the indices used in Cremers, Petajisto, and Zitzewitz’s model are from Morningstar Direct.



**Table 2**  
**Standard Deviations by Quintile of  $R^2$**

This table displays the relationship between the number of funds held in a portfolio and the time-series standard deviation of the returns on the portfolio. Portfolios of 1, 5, 10, 15, 25, 50, 100, and 200 funds are examined. The holding period is from December 1997 through March 2013. The standard deviations are averaged across 1000 simulations. The portfolios equally allocate capital to randomly selected funds that rank in a particular quintile of  $R^2$ . The portfolios are reconstituted at the beginning of each month. Panel A conveys the results from the use of the Carhart (1997) four-factor model to estimate  $R^2$ . Panel B conveys the results from the use of Cremers, Petajisto, and Zitzewitz's (2013) alternative index-based model to estimate  $R^2$ . The  $R^2$  estimation period is 24 months. The returns on the research factors and the risk-free rate are from Kenneth French's website. The returns on mutual funds and the indices used in Cremers, Petajisto, and Zitzewitz's model are from Morningstar Direct.

*Panel A: Carhart Model Results*

Portfolio	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
Quintile 1 Funds	20.67%	17.88%	17.52%	17.39%	17.28%	17.21%	17.17%	17.19%
Quintile 2 Funds	20.20%	18.24%	17.96%	17.88%	17.81%	17.76%	17.72%	17.71%
Quintile 3 Funds	19.84%	18.07%	17.84%	17.77%	17.70%	17.66%	17.64%	17.63%
Quintile 4 Funds	19.46%	18.00%	17.79%	17.73%	17.67%	17.63%	17.61%	17.65%
Quintile 5 Funds	18.44%	17.42%	17.28%	17.24%	17.20%	17.17%	17.16%	17.20%
All Active Funds	19.79%	17.87%	17.62%	17.54%	17.47%	17.41%	17.38%	17.37%
All Passive Funds	18.20%	17.37%	17.27%	17.24%	17.20%	17.18%		

*Panel B: Cremers, Petajisto, and Zitzewitz Model Results*

Portfolio	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
Quintile 1 Funds	20.70%	17.92%	17.54%	17.41%	17.31%	17.23%	17.19%	17.17%
Quintile 2 Funds	20.17%	18.29%	18.05%	17.96%	17.89%	17.85%	17.83%	17.86%
Quintile 3 Funds	19.92%	18.19%	17.96%	17.87%	17.80%	17.75%	17.73%	17.72%
Quintile 4 Funds	19.51%	17.99%	17.78%	17.72%	17.66%	17.62%	17.60%	17.59%
Quintile 5 Funds	18.18%	17.17%	17.05%	17.00%	16.96%	16.94%	16.92%	16.96%
All Active Funds	19.79%	17.87%	17.62%	17.54%	17.47%	17.41%	17.38%	17.37%
All Passive Funds	18.20%	17.37%	17.27%	17.24%	17.20%	17.18%		

**Table 3**  
**Returns by Quintile of  $R^2$  – Carhart Model Results**

This table displays select percentile values of average annualized returns on portfolios that equally allocate capital to randomly selected funds that rank in a particular quintile of  $R^2$  and reconstitute the portfolios at the beginning of each month. Portfolios of 1, 5, 10, 15, 25, 50, 100, and 200 funds are examined. The holding period is from December 1997 through March 2013. Panels A, B, C, D, and E pertain to funds in the bottom, 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, and top quintile of  $R^2$ . Panel F pertains to the entire universe of actively managed funds. Panel G pertains to the entire universe of index funds. The  $R^2$  estimation period is 24 months. The returns on the research factors and the risk-free rate are from Kenneth French’s website. The returns on mutual funds are from Morningstar Direct.

*Panel A: Quintile 1 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	2.57%	6.62%	7.65%	8.14%	8.44%	8.99%	9.31%	9.73%
5%	4.58%	7.76%	8.32%	8.60%	8.90%	9.26%	9.48%	9.80%
10%	5.75%	8.17%	8.70%	8.89%	9.10%	9.39%	9.57%	9.84%
25%	7.70%	8.92%	9.25%	9.34%	9.50%	9.61%	9.72%	9.91%
50%	9.92%	9.92%	9.93%	9.90%	9.92%	9.91%	9.91%	10.00%
75%	11.95%	10.87%	10.62%	10.46%	10.36%	10.19%	10.07%	10.09%
90%	13.95%	11.79%	11.24%	10.95%	10.75%	10.44%	10.25%	10.16%
95%	15.16%	12.38%	11.58%	11.31%	10.96%	10.61%	10.35%	10.21%
99%	17.52%	13.38%	12.56%	12.00%	11.34%	10.83%	10.55%	10.30%

*Panel B: Quintile 2 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	2.18%	5.87%	6.65%	7.02%	7.30%	7.79%	8.10%	8.38%
5%	3.93%	6.62%	7.24%	7.47%	7.76%	8.01%	8.26%	8.48%
10%	5.10%	7.04%	7.49%	7.73%	7.99%	8.17%	8.36%	8.51%
25%	6.70%	7.76%	8.02%	8.19%	8.27%	8.40%	8.49%	8.57%
50%	8.33%	8.59%	8.59%	8.62%	8.60%	8.63%	8.64%	8.63%
75%	10.18%	9.39%	9.19%	9.04%	8.94%	8.85%	8.79%	8.71%
90%	11.80%	10.07%	9.77%	9.44%	9.27%	9.06%	8.94%	8.77%
95%	12.82%	10.49%	10.02%	9.68%	9.45%	9.19%	9.02%	8.81%
99%	14.63%	11.51%	10.56%	10.23%	9.80%	9.37%	9.17%	8.89%

*Panel C: Quintile 3 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	1.76%	5.09%	5.81%	6.14%	6.43%	6.88%	7.08%	7.40%
5%	3.75%	5.67%	6.38%	6.57%	6.83%	7.09%	7.28%	7.47%
10%	4.46%	6.18%	6.63%	6.83%	7.01%	7.21%	7.36%	7.50%
25%	6.03%	6.88%	7.07%	7.16%	7.29%	7.40%	7.48%	7.56%
50%	7.77%	7.66%	7.60%	7.59%	7.61%	7.60%	7.62%	7.62%
75%	9.32%	8.39%	8.13%	8.06%	7.95%	7.83%	7.75%	7.68%
90%	10.90%	9.04%	8.59%	8.47%	8.24%	8.01%	7.86%	7.74%
95%	12.05%	9.50%	8.98%	8.72%	8.41%	8.12%	7.94%	7.77%
99%	14.48%	10.25%	9.47%	9.07%	8.78%	8.35%	8.04%	7.84%

*(Continued)*

**Table 3 - Continued**

<i>Panel D: Quintile 4 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	0.68%	3.63%	4.33%	4.77%	5.09%	5.41%	5.60%	5.82%
5%	2.52%	4.40%	4.88%	5.05%	5.38%	5.61%	5.78%	5.88%
10%	3.29%	4.76%	5.13%	5.26%	5.48%	5.73%	5.84%	5.91%
25%	4.72%	5.32%	5.56%	5.61%	5.74%	5.88%	5.96%	5.96%
50%	6.18%	6.09%	6.05%	6.04%	6.04%	6.07%	6.07%	6.02%
75%	7.67%	6.74%	6.51%	6.41%	6.32%	6.26%	6.20%	6.07%
90%	8.90%	7.32%	6.94%	6.78%	6.61%	6.44%	6.32%	6.13%
95%	9.56%	7.69%	7.15%	6.99%	6.77%	6.55%	6.38%	6.16%
99%	10.87%	8.26%	7.65%	7.45%	7.08%	6.73%	6.51%	6.22%
<i>Panel E: Quintile 5 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	1.31%	3.78%	4.27%	4.44%	4.68%	4.99%	5.15%	5.27%
5%	2.62%	4.21%	4.56%	4.73%	4.93%	5.12%	5.28%	5.33%
10%	3.30%	4.45%	4.76%	4.88%	5.07%	5.21%	5.32%	5.36%
25%	4.36%	4.94%	5.12%	5.22%	5.27%	5.35%	5.41%	5.40%
50%	5.56%	5.51%	5.51%	5.51%	5.49%	5.51%	5.51%	5.44%
75%	6.84%	6.04%	5.89%	5.83%	5.75%	5.67%	5.61%	5.49%
90%	7.94%	6.58%	6.23%	6.11%	5.97%	5.81%	5.71%	5.53%
95%	8.70%	6.83%	6.43%	6.27%	6.10%	5.90%	5.76%	5.56%
99%	10.13%	7.39%	6.83%	6.67%	6.34%	6.07%	5.86%	5.61%
<i>Panel F: All Active Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	1.28%	4.75%	5.57%	5.80%	6.10%	6.47%	6.79%	6.92%
5%	3.45%	5.38%	6.02%	6.21%	6.50%	6.74%	6.93%	7.06%
10%	4.14%	5.86%	6.29%	6.52%	6.69%	6.90%	7.04%	7.13%
25%	5.37%	6.50%	6.79%	6.91%	7.02%	7.10%	7.18%	7.25%
50%	7.27%	7.33%	7.29%	7.38%	7.37%	7.37%	7.36%	7.35%
75%	9.06%	8.17%	7.86%	7.79%	7.70%	7.61%	7.52%	7.47%
90%	10.82%	8.87%	8.43%	8.19%	8.04%	7.84%	7.69%	7.57%
95%	11.81%	9.28%	8.73%	8.53%	8.22%	7.95%	7.78%	7.64%
99%	13.86%	10.21%	9.25%	8.97%	8.60%	8.21%	7.95%	7.77%
<i>Panel G: All Passive Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds		
1%	3.30%	5.73%	6.28%	6.51%	6.67%	6.97%		
5%	4.52%	6.27%	6.58%	6.73%	6.89%	7.09%		
10%	5.20%	6.52%	6.76%	6.89%	7.02%	7.15%		
25%	6.34%	6.91%	7.06%	7.13%	7.19%	7.26%		
50%	7.41%	7.39%	7.36%	7.38%	7.39%	7.39%		
75%	8.50%	7.82%	7.70%	7.64%	7.59%	7.52%		
90%	9.58%	8.29%	8.02%	7.88%	7.75%	7.64%		
95%	10.18%	8.59%	8.23%	8.04%	7.88%	7.71%		
99%	11.73%	9.05%	8.53%	8.26%	8.02%	7.81%		

**Table 4****Returns by Quintile of  $R^2$  – Cremers, Petajisto, and Zitzewitz Model Results**

This table displays select percentile values of average annualized returns on portfolios that equally allocate capital to randomly selected funds that rank in a particular quintile of  $R^2$  and reconstitute the portfolios at the beginning of each month. Portfolios of 1, 5, 10, 15, 25, 50, 100, and 200 funds are examined. The holding period is from December 1997 through March 2013. Panels A, B, C, D, and E pertain to funds in the bottom, 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, and top quintile of  $R^2$ . Panel F pertains to the entire universe of actively managed funds. Panel G pertains to the entire universe of index funds. The  $R^2$  estimation period is 24 months. The returns on mutual funds and the indices used in Cremers, Petajisto, and Zitzewitz's model are from Morningstar Direct.

*Panel A: Quintile 1 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	1.98%	6.83%	7.74%	8.16%	8.63%	9.05%	9.32%	9.63%
5%	4.52%	7.64%	8.32%	8.67%	8.93%	9.27%	9.50%	9.73%
10%	5.99%	8.06%	8.65%	8.95%	9.17%	9.41%	9.60%	9.78%
25%	7.89%	8.95%	9.32%	9.43%	9.52%	9.65%	9.75%	9.84%
50%	9.99%	9.91%	9.98%	9.94%	9.94%	9.92%	9.94%	9.94%
75%	12.20%	10.93%	10.60%	10.45%	10.37%	10.22%	10.10%	10.02%
90%	14.16%	11.84%	11.16%	10.96%	10.72%	10.46%	10.28%	10.09%
95%	15.45%	12.39%	11.51%	11.26%	10.92%	10.59%	10.37%	10.13%
99%	18.11%	13.45%	12.32%	11.88%	11.36%	10.83%	10.53%	10.21%

*Panel B: Quintile 2 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	2.55%	5.99%	6.71%	7.09%	7.58%	7.87%	8.14%	8.53%
5%	4.27%	6.76%	7.33%	7.64%	7.86%	8.09%	8.34%	8.63%
10%	5.25%	7.17%	7.65%	7.84%	8.05%	8.23%	8.42%	8.66%
25%	6.92%	7.96%	8.18%	8.28%	8.36%	8.48%	8.56%	8.73%
50%	8.58%	8.69%	8.68%	8.72%	8.72%	8.70%	8.72%	8.81%
75%	10.30%	9.50%	9.20%	9.15%	9.08%	8.94%	8.85%	8.89%
90%	11.97%	10.18%	9.77%	9.56%	9.36%	9.14%	9.00%	8.95%
95%	13.14%	10.69%	10.16%	9.81%	9.61%	9.26%	9.07%	8.99%
99%	15.55%	11.55%	10.71%	10.32%	9.96%	9.53%	9.23%	9.05%

*Panel C: Quintile 3 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	1.88%	4.79%	5.48%	5.87%	6.22%	6.55%	6.86%	7.13%
5%	3.49%	5.50%	6.02%	6.29%	6.55%	6.82%	7.02%	7.19%
10%	4.29%	5.91%	6.34%	6.54%	6.73%	6.93%	7.09%	7.21%
25%	5.85%	6.59%	6.86%	6.90%	7.02%	7.12%	7.21%	7.27%
50%	7.47%	7.37%	7.34%	7.34%	7.36%	7.33%	7.34%	7.34%
75%	9.08%	8.15%	7.91%	7.81%	7.68%	7.57%	7.49%	7.41%
90%	10.74%	8.87%	8.40%	8.21%	7.96%	7.75%	7.62%	7.47%
95%	11.84%	9.35%	8.66%	8.42%	8.19%	7.88%	7.69%	7.50%
99%	13.86%	10.07%	9.29%	8.85%	8.50%	8.08%	7.85%	7.56%

*(Continued)*



**Table 4 – Continued**

<i>Panel D: Quintile 4 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	0.57%	3.65%	4.54%	4.87%	5.26%	5.53%	5.71%	5.96%
5%	2.42%	4.34%	5.00%	5.19%	5.45%	5.69%	5.87%	6.03%
10%	3.26%	4.72%	5.26%	5.42%	5.64%	5.80%	5.94%	6.06%
25%	4.59%	5.41%	5.65%	5.77%	5.89%	5.99%	6.06%	6.12%
50%	6.18%	6.13%	6.18%	6.18%	6.19%	6.18%	6.18%	6.18%
75%	7.60%	6.82%	6.65%	6.60%	6.48%	6.41%	6.30%	6.24%
90%	9.14%	7.50%	7.08%	6.91%	6.75%	6.57%	6.41%	6.30%
95%	10.10%	7.89%	7.30%	7.13%	6.93%	6.67%	6.49%	6.33%
99%	12.02%	8.44%	7.82%	7.52%	7.17%	6.88%	6.60%	6.38%
<i>Panel E: Quintile 5 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	1.39%	3.85%	4.33%	4.55%	4.79%	5.11%	5.27%	5.36%
5%	2.71%	4.34%	4.71%	4.85%	5.05%	5.23%	5.37%	5.40%
10%	3.33%	4.58%	4.88%	5.02%	5.15%	5.31%	5.42%	5.44%
25%	4.42%	5.07%	5.21%	5.28%	5.37%	5.44%	5.50%	5.47%
50%	5.62%	5.60%	5.59%	5.58%	5.60%	5.60%	5.61%	5.52%
75%	6.83%	6.16%	5.95%	5.88%	5.83%	5.74%	5.70%	5.57%
90%	8.01%	6.65%	6.31%	6.20%	6.04%	5.89%	5.78%	5.61%
95%	8.72%	6.91%	6.49%	6.35%	6.16%	5.98%	5.84%	5.63%
99%	9.88%	7.43%	6.92%	6.66%	6.35%	6.14%	5.91%	5.68%
<i>Panel F: All Active Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	1.28%	4.75%	5.57%	5.80%	6.10%	6.47%	6.79%	6.92%
5%	3.45%	5.38%	6.02%	6.21%	6.50%	6.74%	6.93%	7.06%
10%	4.14%	5.86%	6.29%	6.52%	6.69%	6.90%	7.04%	7.13%
25%	5.37%	6.50%	6.79%	6.91%	7.02%	7.10%	7.18%	7.25%
50%	7.27%	7.33%	7.29%	7.38%	7.37%	7.37%	7.36%	7.35%
75%	9.06%	8.17%	7.86%	7.79%	7.70%	7.61%	7.52%	7.47%
90%	10.82%	8.87%	8.43%	8.19%	8.04%	7.84%	7.69%	7.57%
95%	11.81%	9.28%	8.73%	8.53%	8.22%	7.95%	7.78%	7.64%
99%	13.86%	10.21%	9.25%	8.97%	8.60%	8.21%	7.95%	7.77%
<i>Panel G: All Passive Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds		
1%	3.30%	5.73%	6.28%	6.51%	6.67%	6.97%		
5%	4.52%	6.27%	6.58%	6.73%	6.89%	7.09%		
10%	5.20%	6.52%	6.76%	6.89%	7.02%	7.15%		
25%	6.34%	6.91%	7.06%	7.13%	7.19%	7.26%		
50%	7.41%	7.39%	7.36%	7.38%	7.39%	7.39%		
75%	8.50%	7.82%	7.70%	7.64%	7.59%	7.52%		
90%	9.58%	8.29%	8.02%	7.88%	7.75%	7.64%		
95%	10.18%	8.59%	8.23%	8.04%	7.88%	7.71%		
99%	11.73%	9.05%	8.53%	8.26%	8.02%	7.81%		

**Table 5**

**Ending Wealth Per Dollar Invested by Quintile of  $R^2$  – Carhart Model Results**

This table displays select percentile values of terminal wealth in March 2013 on one dollar invested in December 1997 in portfolios that equally allocate capital to randomly selected funds that rank in a particular quintile of  $R^2$ . The portfolios are reconstituted at the beginning of each month. Portfolios of 1, 5, 10, 15, 25, 50, 100, and 200 funds are examined. Panels A, B, C, D, and E pertain to funds in the bottom, 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, and top quintile of  $R^2$ . Panel F pertains to the entire universe of actively managed funds. Panel G pertains to the entire universe of index funds. The  $R^2$  estimation period is 24 months. The returns on the research factors and the risk-free rate are from Kenneth French’s website. The returns on mutual funds are from Morningstar Direct.

*Panel A: Quintile 1 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$1.02	\$2.05	\$2.44	\$2.64	\$2.75	\$2.97	\$3.12	\$3.25
5%	\$1.38	\$2.45	\$2.68	\$2.81	\$2.94	\$3.10	\$3.19	\$3.28
10%	\$1.72	\$2.62	\$2.83	\$2.93	\$3.03	\$3.15	\$3.23	\$3.30
25%	\$2.26	\$2.91	\$3.06	\$3.11	\$3.19	\$3.25	\$3.30	\$3.34
50%	\$3.06	\$3.33	\$3.36	\$3.36	\$3.38	\$3.39	\$3.39	\$3.38
75%	\$4.08	\$3.80	\$3.70	\$3.64	\$3.60	\$3.52	\$3.47	\$3.42
90%	\$5.25	\$4.31	\$4.04	\$3.89	\$3.79	\$3.64	\$3.55	\$3.45
95%	\$6.31	\$4.69	\$4.22	\$4.09	\$3.92	\$3.73	\$3.60	\$3.48
99%	\$8.68	\$5.35	\$4.82	\$4.45	\$4.12	\$3.87	\$3.70	\$3.52

*Panel B: Quintile 2 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$1.01	\$1.84	\$2.09	\$2.21	\$2.31	\$2.47	\$2.59	\$2.69
5%	\$1.29	\$2.07	\$2.27	\$2.36	\$2.46	\$2.55	\$2.65	\$2.73
10%	\$1.55	\$2.20	\$2.36	\$2.45	\$2.54	\$2.61	\$2.68	\$2.74
25%	\$1.97	\$2.43	\$2.55	\$2.60	\$2.65	\$2.69	\$2.73	\$2.77
50%	\$2.51	\$2.74	\$2.75	\$2.77	\$2.77	\$2.79	\$2.79	\$2.79
75%	\$3.25	\$3.05	\$2.99	\$2.95	\$2.91	\$2.87	\$2.86	\$2.82
90%	\$4.05	\$3.35	\$3.25	\$3.12	\$3.04	\$2.96	\$2.91	\$2.85
95%	\$4.59	\$3.57	\$3.34	\$3.21	\$3.11	\$3.01	\$2.94	\$2.86
99%	\$5.65	\$4.03	\$3.64	\$3.47	\$3.27	\$3.10	\$3.00	\$2.89

*Panel C: Quintile 3 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.95	\$1.64	\$1.86	\$1.95	\$2.04	\$2.17	\$2.25	\$2.35
5%	\$1.29	\$1.81	\$2.01	\$2.08	\$2.16	\$2.24	\$2.31	\$2.37
10%	\$1.44	\$1.94	\$2.09	\$2.15	\$2.22	\$2.28	\$2.34	\$2.38
25%	\$1.82	\$2.16	\$2.23	\$2.26	\$2.31	\$2.35	\$2.38	\$2.40
50%	\$2.32	\$2.42	\$2.40	\$2.41	\$2.41	\$2.42	\$2.42	\$2.42
75%	\$2.93	\$2.67	\$2.59	\$2.57	\$2.54	\$2.49	\$2.47	\$2.44
90%	\$3.61	\$2.91	\$2.78	\$2.72	\$2.64	\$2.56	\$2.51	\$2.46
95%	\$4.20	\$3.12	\$2.91	\$2.82	\$2.70	\$2.60	\$2.54	\$2.48
99%	\$5.61	\$3.46	\$3.12	\$2.95	\$2.86	\$2.68	\$2.57	\$2.50

(Continued)

**Table 5 - Continued**

<i>Panel D: Quintile 4 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.82	\$1.32	\$1.49	\$1.59	\$1.68	\$1.76	\$1.82	\$1.86
5%	\$1.06	\$1.50	\$1.63	\$1.67	\$1.75	\$1.81	\$1.86	\$1.88
10%	\$1.23	\$1.58	\$1.68	\$1.72	\$1.78	\$1.84	\$1.88	\$1.89
25%	\$1.50	\$1.72	\$1.79	\$1.81	\$1.85	\$1.89	\$1.91	\$1.90
50%	\$1.87	\$1.92	\$1.93	\$1.92	\$1.93	\$1.94	\$1.94	\$1.92
75%	\$2.32	\$2.11	\$2.06	\$2.03	\$2.01	\$1.99	\$1.98	\$1.93
90%	\$2.76	\$2.30	\$2.19	\$2.15	\$2.09	\$2.04	\$2.01	\$1.95
95%	\$3.03	\$2.42	\$2.26	\$2.21	\$2.15	\$2.08	\$2.03	\$1.96
99%	\$3.66	\$2.65	\$2.42	\$2.36	\$2.23	\$2.14	\$2.07	\$1.97
<i>Panel E: Quintile 5 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.92	\$1.38	\$1.50	\$1.53	\$1.60	\$1.67	\$1.72	\$1.74
5%	\$1.12	\$1.47	\$1.57	\$1.61	\$1.66	\$1.71	\$1.75	\$1.76
10%	\$1.24	\$1.54	\$1.62	\$1.65	\$1.69	\$1.73	\$1.76	\$1.76
25%	\$1.47	\$1.66	\$1.70	\$1.73	\$1.75	\$1.77	\$1.78	\$1.77
50%	\$1.76	\$1.80	\$1.81	\$1.81	\$1.80	\$1.81	\$1.81	\$1.79
75%	\$2.13	\$1.95	\$1.91	\$1.89	\$1.87	\$1.85	\$1.84	\$1.80
90%	\$2.48	\$2.10	\$2.01	\$1.97	\$1.94	\$1.89	\$1.86	\$1.81
95%	\$2.76	\$2.18	\$2.07	\$2.02	\$1.97	\$1.92	\$1.88	\$1.82
99%	\$3.36	\$2.36	\$2.18	\$2.14	\$2.05	\$1.97	\$1.91	\$1.83
<i>Panel F: All Active Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.88	\$1.57	\$1.81	\$1.88	\$1.97	\$2.07	\$2.16	\$2.21
5%	\$1.22	\$1.75	\$1.92	\$1.99	\$2.08	\$2.15	\$2.21	\$2.25
10%	\$1.38	\$1.87	\$2.01	\$2.08	\$2.13	\$2.20	\$2.25	\$2.28
25%	\$1.66	\$2.05	\$2.15	\$2.19	\$2.23	\$2.26	\$2.29	\$2.31
50%	\$2.18	\$2.31	\$2.32	\$2.35	\$2.35	\$2.35	\$2.35	\$2.35
75%	\$2.79	\$2.60	\$2.51	\$2.49	\$2.46	\$2.44	\$2.40	\$2.39
90%	\$3.60	\$2.87	\$2.71	\$2.63	\$2.59	\$2.51	\$2.46	\$2.42
95%	\$4.06	\$3.03	\$2.84	\$2.75	\$2.65	\$2.56	\$2.49	\$2.45
99%	\$5.37	\$3.42	\$3.04	\$2.94	\$2.80	\$2.65	\$2.55	\$2.49
<i>Panel G: All Passive Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds		
1%	\$1.26	\$1.86	\$2.02	\$2.08	\$2.14	\$2.23		
5%	\$1.51	\$2.01	\$2.11	\$2.16	\$2.20	\$2.27		
10%	\$1.68	\$2.08	\$2.16	\$2.21	\$2.25	\$2.29		
25%	\$1.99	\$2.21	\$2.26	\$2.28	\$2.31	\$2.33		
50%	\$2.32	\$2.36	\$2.36	\$2.37	\$2.38	\$2.37		
75%	\$2.71	\$2.52	\$2.48	\$2.45	\$2.44	\$2.42		
90%	\$3.14	\$2.69	\$2.59	\$2.55	\$2.50	\$2.46		
95%	\$3.42	\$2.81	\$2.66	\$2.61	\$2.54	\$2.48		
99%	\$4.24	\$2.97	\$2.78	\$2.68	\$2.60	\$2.52		

**Table 6****Ending Wealth Per Dollar Invested by Quintile of  $R^2$  – Cremers, Petajisto, and Zitzewitz Model Results**

This table displays select percentile values of terminal wealth in March 2013 on one dollar invested in December 1997 in portfolios that equally allocate capital to randomly selected funds that rank in a particular quintile of  $R^2$ . The portfolios are reconstituted at the beginning of each month. Portfolios of 1, 5, 10, 15, 25, 50, 100, and 200 funds are examined. Panels A, B, C, D, and E pertain to funds in the bottom, 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, and top quintile of  $R^2$ . Panel F pertains to the entire universe of actively managed funds. Panel G pertains to the entire universe of index funds. The  $R^2$  estimation period is 24 months. The returns on mutual funds and the indices used in Cremers, Petajisto, and Zitzewitz's model are from Morningstar Direct.

*Panel A: Quintile 1 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.95	\$2.15	\$2.47	\$2.64	\$2.81	\$2.99	\$3.12	\$3.26
5%	\$1.39	\$2.41	\$2.68	\$2.83	\$2.94	\$3.09	\$3.20	\$3.31
10%	\$1.72	\$2.56	\$2.81	\$2.95	\$3.05	\$3.16	\$3.24	\$3.33
25%	\$2.29	\$2.91	\$3.10	\$3.16	\$3.19	\$3.27	\$3.31	\$3.36
50%	\$3.13	\$3.32	\$3.39	\$3.38	\$3.40	\$3.39	\$3.40	\$3.40
75%	\$4.15	\$3.82	\$3.70	\$3.63	\$3.60	\$3.53	\$3.48	\$3.44
90%	\$5.44	\$4.30	\$4.00	\$3.89	\$3.78	\$3.65	\$3.57	\$3.48
95%	\$6.46	\$4.65	\$4.21	\$4.05	\$3.88	\$3.72	\$3.61	\$3.50
99%	\$9.29	\$5.35	\$4.60	\$4.37	\$4.12	\$3.84	\$3.68	\$3.54

*Panel B: Quintile 2 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$1.09	\$1.90	\$2.11	\$2.24	\$2.39	\$2.49	\$2.60	\$2.70
5%	\$1.36	\$2.10	\$2.31	\$2.41	\$2.49	\$2.58	\$2.67	\$2.73
10%	\$1.60	\$2.24	\$2.40	\$2.48	\$2.56	\$2.63	\$2.70	\$2.75
25%	\$2.04	\$2.50	\$2.60	\$2.63	\$2.68	\$2.72	\$2.75	\$2.78
50%	\$2.59	\$2.76	\$2.79	\$2.80	\$2.81	\$2.81	\$2.82	\$2.80
75%	\$3.30	\$3.10	\$3.00	\$2.98	\$2.95	\$2.90	\$2.87	\$2.83
90%	\$4.20	\$3.42	\$3.26	\$3.16	\$3.08	\$2.99	\$2.93	\$2.86
95%	\$4.83	\$3.64	\$3.42	\$3.27	\$3.18	\$3.04	\$2.96	\$2.87
99%	\$6.53	\$4.13	\$3.71	\$3.50	\$3.35	\$3.14	\$3.02	\$2.90

*Panel C: Quintile 3 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.97	\$1.58	\$1.76	\$1.87	\$1.97	\$2.08	\$2.17	\$2.25
5%	\$1.21	\$1.76	\$1.90	\$1.99	\$2.07	\$2.16	\$2.22	\$2.27
10%	\$1.37	\$1.87	\$2.00	\$2.06	\$2.13	\$2.19	\$2.24	\$2.28
25%	\$1.76	\$2.06	\$2.15	\$2.18	\$2.21	\$2.25	\$2.28	\$2.30
50%	\$2.23	\$2.31	\$2.31	\$2.32	\$2.32	\$2.32	\$2.32	\$2.32
75%	\$2.79	\$2.58	\$2.50	\$2.47	\$2.43	\$2.40	\$2.37	\$2.35
90%	\$3.51	\$2.85	\$2.68	\$2.62	\$2.53	\$2.46	\$2.42	\$2.37
95%	\$4.08	\$3.04	\$2.79	\$2.70	\$2.61	\$2.51	\$2.44	\$2.38
99%	\$5.29	\$3.33	\$3.03	\$2.85	\$2.74	\$2.58	\$2.49	\$2.40

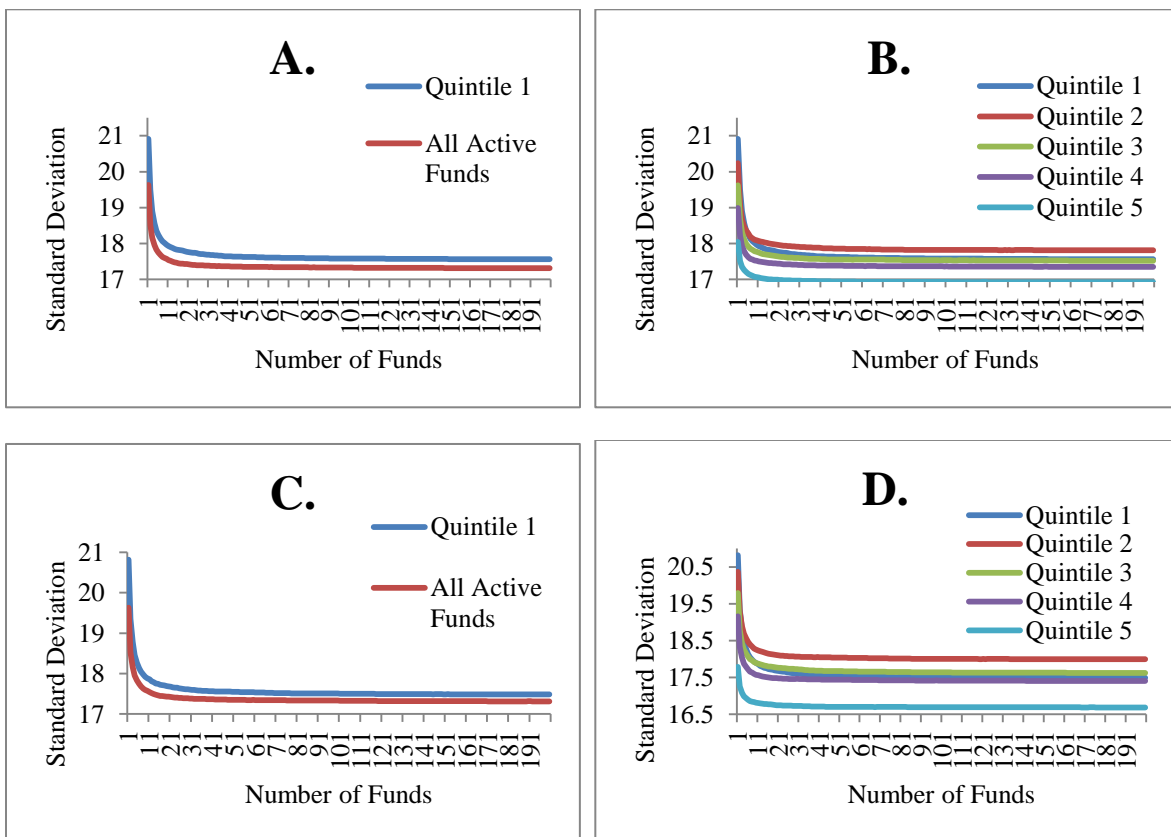
*(Continued)*

**Table 6 - Continued**

<i>Panel D: Quintile 4 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.82	\$1.34	\$1.55	\$1.62	\$1.71	\$1.79	\$1.84	\$1.91
5%	\$1.07	\$1.48	\$1.65	\$1.70	\$1.77	\$1.84	\$1.89	\$1.93
10%	\$1.21	\$1.58	\$1.71	\$1.76	\$1.81	\$1.86	\$1.90	\$1.94
25%	\$1.49	\$1.75	\$1.82	\$1.85	\$1.89	\$1.92	\$1.94	\$1.95
50%	\$1.88	\$1.94	\$1.96	\$1.97	\$1.97	\$1.97	\$1.97	\$1.97
75%	\$2.30	\$2.14	\$2.10	\$2.09	\$2.06	\$2.04	\$2.01	\$1.99
90%	\$2.84	\$2.36	\$2.23	\$2.19	\$2.14	\$2.08	\$2.04	\$2.01
95%	\$3.28	\$2.48	\$2.32	\$2.25	\$2.19	\$2.12	\$2.06	\$2.02
99%	\$4.16	\$2.69	\$2.48	\$2.39	\$2.28	\$2.18	\$2.09	\$2.03
<i>Panel E: Quintile 5 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.96	\$1.42	\$1.53	\$1.58	\$1.64	\$1.72	\$1.76	\$1.78
5%	\$1.15	\$1.52	\$1.61	\$1.65	\$1.70	\$1.75	\$1.79	\$1.79
10%	\$1.27	\$1.58	\$1.65	\$1.69	\$1.73	\$1.77	\$1.80	\$1.80
25%	\$1.50	\$1.70	\$1.74	\$1.76	\$1.78	\$1.80	\$1.82	\$1.81
50%	\$1.79	\$1.84	\$1.84	\$1.84	\$1.84	\$1.85	\$1.85	\$1.82
75%	\$2.12	\$1.99	\$1.94	\$1.92	\$1.91	\$1.89	\$1.87	\$1.83
90%	\$2.51	\$2.14	\$2.04	\$2.01	\$1.97	\$1.93	\$1.90	\$1.84
95%	\$2.80	\$2.23	\$2.10	\$2.06	\$2.00	\$1.95	\$1.91	\$1.85
99%	\$3.34	\$2.42	\$2.23	\$2.15	\$2.06	\$2.00	\$1.94	\$1.86
<i>Panel F: All Active Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.88	\$1.57	\$1.81	\$1.88	\$1.97	\$2.07	\$2.16	\$2.21
5%	\$1.22	\$1.75	\$1.92	\$1.99	\$2.08	\$2.15	\$2.21	\$2.25
10%	\$1.38	\$1.87	\$2.01	\$2.08	\$2.13	\$2.20	\$2.25	\$2.28
25%	\$1.66	\$2.05	\$2.15	\$2.19	\$2.23	\$2.26	\$2.29	\$2.31
50%	\$2.18	\$2.31	\$2.32	\$2.35	\$2.35	\$2.35	\$2.35	\$2.35
75%	\$2.79	\$2.60	\$2.51	\$2.49	\$2.46	\$2.44	\$2.40	\$2.39
90%	\$3.60	\$2.87	\$2.71	\$2.63	\$2.59	\$2.51	\$2.46	\$2.42
95%	\$4.06	\$3.03	\$2.84	\$2.75	\$2.65	\$2.56	\$2.49	\$2.45
99%	\$5.37	\$3.42	\$3.04	\$2.94	\$2.80	\$2.65	\$2.55	\$2.49
<i>Panel G: All Passive Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds		
1%	\$1.26	\$1.86	\$2.02	\$2.08	\$2.14	\$2.23		
5%	\$1.51	\$2.01	\$2.11	\$2.16	\$2.20	\$2.27		
10%	\$1.68	\$2.08	\$2.16	\$2.21	\$2.25	\$2.29		
25%	\$1.99	\$2.21	\$2.26	\$2.28	\$2.31	\$2.33		
50%	\$2.32	\$2.36	\$2.36	\$2.37	\$2.38	\$2.37		
75%	\$2.71	\$2.52	\$2.48	\$2.45	\$2.44	\$2.42		
90%	\$3.14	\$2.69	\$2.59	\$2.55	\$2.50	\$2.46		
95%	\$3.42	\$2.81	\$2.66	\$2.61	\$2.54	\$2.48		
99%	\$4.24	\$2.97	\$2.78	\$2.68	\$2.60	\$2.52		

**Figure 3**  
**Number of Funds and Standard Deviation**

These graphics display the relationship between the number of funds held in a portfolio and the time-series standard deviation of the returns on the portfolio. The holding period is from December 1997 through March 2013. The standard deviations are averaged across 1000 simulations. The portfolios equally allocate capital to randomly selected funds that rank in a particular quintile of  $R^2$ . The portfolios are reconstituted every 12 months. Graphics A and C plot this relationship for a portfolio of funds ranked in the bottom quintile of  $R^2$  and a “All Active Funds” portfolio, consisting of all of the funds that comprise the  $R^2$ -sorted portfolios. Graphic B and D plot this relationship for funds ranked in each quintile of  $R^2$ . Graphics A and B use the Carhart (1997) four-factor model to estimate  $R^2$ . Graphics C and D use Cremers, Petajisto, and Zitzewitz’s (2013) alternative index-based model to estimate  $R^2$ . The  $R^2$  estimation period is 24 months. The returns on the research factors and the risk-free rate are from Kenneth French’s website. The returns on mutual funds and the indices used in Cremers, Petajisto, and Zitzewitz’s model are from Morningstar Direct.



**Table 7**  
**Standard Deviations by Quintile of  $R^2$**

This table displays the relationship between the number of funds held in a portfolio and the time-series standard deviation of the returns on the portfolio. Portfolios of 1, 5, 10, 15, 25, 50, 100, and 200 funds are examined. The holding period is from December 1997 through March 2013. The standard deviations are averaged across 1000 simulations. The portfolios equally allocate capital to randomly selected funds that rank in a particular quintile of  $R^2$ . The portfolios are reconstituted every 12 months. Panel A conveys the results from the use of the Carhart (1997) four-factor model to estimate  $R^2$ . Panel B conveys the results from the use of Cremers, Petajisto, and Zitzewitz's (2013) alternative index-based model to estimate  $R^2$ . The  $R^2$  estimation period is 24 months. The returns on the research factors and the risk-free rate are from Kenneth French's website. The returns on mutual funds and the indices used in Cremers, Petajisto, and Zitzewitz's model are from Morningstar Direct.

*Panel A: Carhart Model Results*

Portfolio	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
Quintile 1 Funds	20.91%	18.36%	17.96%	17.83%	17.73%	17.63%	17.58%	17.57%
Quintile 2 Funds	20.23%	18.36%	18.09%	18.02%	17.93%	17.86%	17.82%	17.81%
Quintile 3 Funds	19.62%	17.99%	17.76%	17.69%	17.61%	17.55%	17.53%	17.52%
Quintile 4 Funds	18.99%	17.69%	17.52%	17.47%	17.42%	17.38%	17.36%	17.35%
Quintile 5 Funds	18.04%	17.21%	17.06%	17.01%	16.97%	16.94%	16.93%	16.92%
All Active Funds	19.63%	17.83%	17.57%	17.46%	17.40%	17.35%	17.33%	17.31%
All Passive Funds	17.90%	17.20%	17.10%	17.07%	17.05%	17.03%		

*Panel B: Cremers, Petajisto, and Zitzewitz Model Results*

Portfolio	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
Quintile 1 Funds	20.82%	18.27%	17.88%	17.75%	17.64%	17.55%	17.51%	17.49%
Quintile 2 Funds	20.37%	18.56%	18.25%	18.16%	18.08%	18.04%	18.00%	17.99%
Quintile 3 Funds	19.79%	18.14%	17.89%	17.81%	17.74%	17.67%	17.64%	17.62%
Quintile 4 Funds	19.16%	17.78%	17.57%	17.51%	17.46%	17.43%	17.41%	17.40%
Quintile 5 Funds	17.78%	16.93%	16.81%	16.77%	16.73%	16.70%	16.69%	16.68%
All Active Funds	19.63%	17.83%	17.57%	17.46%	17.40%	17.35%	17.33%	17.31%
All Passive Funds	17.90%	17.20%	17.10%	17.07%	17.05%	17.03%		

**Table 8**  
**Returns by Quintile of  $R^2$  – Carhart Model Results**

This table displays select percentile values of average annualized returns on portfolios that equally allocate capital to randomly selected funds that rank in a particular quintile of  $R^2$  and reconstitute the portfolios every 12 months. Portfolios of 1, 5, 10, 15, 25, 50, 100, and 200 funds are examined. The holding period is from December 1997 through March 2013. Panels A, B, C, D, and E pertain to funds in the bottom, 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, and top quintile of  $R^2$ . Panel F pertains to the entire universe of actively managed funds. Panel G pertains to the entire universe of index funds. The  $R^2$  estimation period is 24 months. The returns on the research factors and the risk-free rate are from Kenneth French's website. The returns on mutual funds are from Morningstar Direct.

*Panel A: Quintile 1 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	-1.32%	5.32%	6.55%	7.24%	7.62%	8.27%	8.64%	9.02%
5%	3.00%	6.67%	7.38%	7.85%	8.11%	8.53%	8.88%	9.14%
10%	4.48%	7.18%	7.78%	8.13%	8.43%	8.75%	9.00%	9.21%
25%	6.56%	8.11%	8.57%	8.72%	8.89%	9.09%	9.19%	9.32%
50%	9.04%	9.30%	9.38%	9.37%	9.43%	9.42%	9.45%	9.45%
75%	11.47%	10.46%	10.24%	10.06%	9.98%	9.80%	9.68%	9.59%
90%	13.79%	11.58%	11.02%	10.70%	10.47%	10.14%	9.90%	9.72%
95%	15.02%	12.36%	11.42%	11.16%	10.74%	10.33%	10.01%	9.78%
99%	17.55%	13.86%	12.46%	12.00%	11.24%	10.69%	10.26%	9.87%

*Panel B: Quintile 2 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	0.77%	4.85%	5.72%	6.30%	6.68%	7.18%	7.41%	7.68%
5%	3.37%	5.72%	6.36%	6.72%	7.05%	7.37%	7.60%	7.77%
10%	4.31%	6.21%	6.70%	7.00%	7.25%	7.50%	7.69%	7.82%
25%	6.01%	7.06%	7.30%	7.42%	7.59%	7.73%	7.82%	7.90%
50%	7.78%	7.93%	8.01%	7.99%	7.98%	8.00%	8.00%	8.00%
75%	9.69%	8.89%	8.63%	8.50%	8.35%	8.25%	8.16%	8.09%
90%	11.44%	9.67%	9.20%	8.93%	8.70%	8.48%	8.31%	8.17%
95%	12.55%	10.24%	9.54%	9.18%	8.92%	8.61%	8.39%	8.21%
99%	14.57%	11.32%	10.18%	9.69%	9.30%	8.90%	8.59%	8.30%

*Panel C: Quintile 3 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	1.21%	4.77%	5.38%	5.84%	6.26%	6.63%	6.83%	7.12%
5%	3.29%	5.49%	6.00%	6.27%	6.55%	6.83%	7.04%	7.22%
10%	4.22%	5.93%	6.37%	6.56%	6.74%	6.96%	7.12%	7.26%
25%	5.81%	6.66%	6.84%	6.96%	7.05%	7.18%	7.27%	7.35%
50%	7.47%	7.43%	7.44%	7.39%	7.43%	7.44%	7.44%	7.43%
75%	9.37%	8.23%	8.01%	7.95%	7.80%	7.69%	7.58%	7.52%
90%	11.18%	9.02%	8.53%	8.37%	8.11%	7.90%	7.71%	7.59%
95%	12.15%	9.52%	8.82%	8.54%	8.29%	8.00%	7.80%	7.63%
99%	13.79%	10.23%	9.27%	9.07%	8.62%	8.22%	7.90%	7.75%

*(Continued)*



**Table 8 - Continued**

<i>Panel D: Quintile 4 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	1.20%	4.14%	4.87%	5.00%	5.35%	5.64%	5.94%	6.09%
5%	2.69%	4.70%	5.18%	5.50%	5.65%	5.86%	6.04%	6.18%
10%	3.70%	5.11%	5.44%	5.66%	5.81%	5.96%	6.10%	6.22%
25%	4.90%	5.70%	5.89%	6.00%	6.07%	6.15%	6.22%	6.28%
50%	6.27%	6.34%	6.35%	6.37%	6.37%	6.33%	6.35%	6.35%
75%	7.84%	7.05%	6.86%	6.76%	6.67%	6.53%	6.47%	6.42%
90%	9.35%	7.76%	7.26%	7.11%	6.91%	6.72%	6.59%	6.48%
95%	10.22%	8.08%	7.52%	7.23%	7.02%	6.83%	6.66%	6.53%
99%	11.75%	8.74%	8.06%	7.61%	7.38%	7.02%	6.77%	6.60%
<i>Panel E: Quintile 5 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	1.10%	3.67%	4.25%	4.59%	4.82%	5.05%	5.22%	5.39%
5%	2.67%	4.17%	4.66%	4.84%	5.04%	5.20%	5.35%	5.46%
10%	3.31%	4.52%	4.88%	5.01%	5.15%	5.30%	5.41%	5.49%
25%	4.49%	5.06%	5.23%	5.29%	5.36%	5.44%	5.50%	5.54%
50%	5.54%	5.57%	5.58%	5.59%	5.59%	5.60%	5.60%	5.60%
75%	6.64%	6.12%	5.95%	5.90%	5.83%	5.77%	5.69%	5.64%
90%	7.61%	6.55%	6.31%	6.18%	6.02%	5.90%	5.78%	5.70%
95%	8.53%	6.84%	6.49%	6.31%	6.15%	5.98%	5.84%	5.72%
99%	9.76%	7.51%	6.85%	6.61%	6.40%	6.16%	5.93%	5.79%
<i>Panel F: All Active Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	0.97%	4.64%	5.27%	5.51%	5.91%	6.36%	6.71%	6.89%
5%	2.90%	5.31%	5.83%	6.12%	6.42%	6.71%	6.90%	7.03%
10%	3.98%	5.78%	6.19%	6.33%	6.64%	6.86%	7.01%	7.12%
25%	5.59%	6.44%	6.75%	6.80%	6.98%	7.11%	7.19%	7.24%
50%	7.13%	7.27%	7.28%	7.34%	7.36%	7.37%	7.38%	7.37%
75%	8.84%	8.10%	7.94%	7.87%	7.74%	7.65%	7.57%	7.50%
90%	10.69%	9.03%	8.57%	8.35%	8.13%	7.89%	7.73%	7.63%
95%	11.80%	9.51%	8.94%	8.70%	8.35%	8.06%	7.84%	7.71%
99%	14.85%	10.90%	9.78%	9.20%	8.66%	8.31%	8.06%	7.80%
<i>Panel G: All Passive Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds		
1%	4.07%	5.90%	6.39%	6.48%	6.79%	7.08%		
5%	5.03%	6.28%	6.67%	6.80%	6.97%	7.18%		
10%	5.58%	6.57%	6.86%	6.97%	7.09%	7.23%		
25%	6.40%	6.97%	7.13%	7.19%	7.27%	7.33%		
50%	7.34%	7.44%	7.45%	7.45%	7.44%	7.44%		
75%	8.38%	7.94%	7.77%	7.71%	7.64%	7.55%		
90%	9.31%	8.42%	8.09%	7.92%	7.80%	7.65%		
95%	10.15%	8.72%	8.29%	8.07%	7.90%	7.70%		
99%	11.69%	9.07%	8.56%	8.33%	8.10%	7.81%		

**Table 9****Returns by Quintile of  $R^2$  – Cremers, Petajisto, and Zitzewitz Model Results**

This table displays select percentile values of average annualized returns on portfolios that equally allocate capital to randomly selected funds that rank in a particular quintile of  $R^2$  and reconstitute the portfolios every 12 months. Portfolios of 1, 5, 10, 15, 25, 50, 100, and 200 funds are examined. The holding period is from December 1997 through March 2013. Panels A, B, C, D, and E pertain to funds in the bottom, 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, and top quintile of  $R^2$ . Panel F pertains to the entire universe of actively managed funds. Panel G pertains to the entire universe of index funds. The  $R^2$  estimation period is 24 months. The returns on mutual funds and the indices used in Cremers, Petajisto, and Zitzewitz's model are from Morningstar Direct.

*Panel A: Quintile 1 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	0.98%	5.47%	6.76%	7.01%	7.57%	8.15%	8.55%	8.85%
5%	3.63%	6.75%	7.44%	7.67%	8.10%	8.53%	8.77%	9.01%
10%	5.00%	7.19%	7.85%	8.02%	8.37%	8.70%	8.91%	9.08%
25%	7.06%	8.22%	8.55%	8.65%	8.83%	8.99%	9.11%	9.19%
50%	9.39%	9.37%	9.37%	9.33%	9.33%	9.32%	9.32%	9.33%
75%	11.71%	10.57%	10.24%	10.00%	9.92%	9.67%	9.54%	9.44%
90%	14.24%	11.55%	10.96%	10.64%	10.28%	10.00%	9.78%	9.56%
95%	15.62%	12.41%	11.50%	11.05%	10.55%	10.21%	9.90%	9.63%
99%	19.02%	14.18%	12.41%	11.72%	11.24%	10.48%	10.13%	9.80%

*Panel B: Quintile 2 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	1.82%	5.17%	6.22%	6.55%	7.04%	7.46%	7.77%	8.01%
5%	3.75%	6.09%	6.80%	7.09%	7.38%	7.67%	7.91%	8.10%
10%	4.66%	6.65%	7.10%	7.39%	7.61%	7.83%	7.99%	8.15%
25%	6.29%	7.38%	7.71%	7.86%	7.91%	8.02%	8.14%	8.24%
50%	8.16%	8.26%	8.34%	8.34%	8.31%	8.32%	8.34%	8.32%
75%	9.93%	9.20%	8.96%	8.85%	8.72%	8.59%	8.50%	8.42%
90%	11.72%	10.04%	9.51%	9.29%	9.09%	8.84%	8.65%	8.51%
95%	12.90%	10.42%	9.90%	9.62%	9.32%	8.96%	8.70%	8.56%
99%	15.13%	11.55%	10.56%	10.10%	9.58%	9.20%	8.90%	8.64%

*Panel C: Quintile 3 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	0.72%	4.37%	5.19%	5.50%	5.89%	6.22%	6.53%	6.74%
5%	2.60%	5.08%	5.72%	5.97%	6.21%	6.50%	6.70%	6.85%
10%	3.87%	5.41%	5.93%	6.19%	6.37%	6.61%	6.77%	6.89%
25%	5.15%	6.08%	6.45%	6.59%	6.69%	6.84%	6.90%	6.97%
50%	6.87%	7.05%	7.03%	7.07%	7.04%	7.07%	7.05%	7.05%
75%	8.58%	7.85%	7.62%	7.53%	7.40%	7.29%	7.20%	7.13%
90%	10.15%	8.55%	8.18%	7.98%	7.70%	7.51%	7.34%	7.20%
95%	11.10%	8.97%	8.48%	8.20%	7.93%	7.66%	7.41%	7.24%
99%	12.81%	9.81%	9.02%	8.74%	8.30%	7.96%	7.58%	7.35%

*(Continued)*

**Table 9 - Continued**

<i>Panel D: Quintile 4 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	1.14%	4.06%	4.70%	4.95%	5.24%	5.63%	5.85%	6.11%
5%	2.68%	4.67%	5.19%	5.30%	5.60%	5.86%	6.03%	6.17%
10%	3.47%	5.10%	5.39%	5.57%	5.74%	5.95%	6.09%	6.21%
25%	4.64%	5.61%	5.86%	5.94%	6.02%	6.14%	6.21%	6.28%
50%	6.24%	6.33%	6.36%	6.35%	6.37%	6.35%	6.34%	6.35%
75%	7.77%	7.01%	6.85%	6.76%	6.65%	6.54%	6.47%	6.42%
90%	9.14%	7.70%	7.27%	7.07%	6.90%	6.72%	6.60%	6.48%
95%	9.95%	8.05%	7.46%	7.30%	7.03%	6.82%	6.65%	6.51%
99%	11.72%	8.55%	7.94%	7.63%	7.30%	7.00%	6.74%	6.58%
<i>Panel E: Quintile 5 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	2.22%	3.98%	4.55%	4.71%	5.02%	5.28%	5.42%	5.59%
5%	3.02%	4.50%	4.88%	5.02%	5.22%	5.39%	5.52%	5.64%
10%	3.67%	4.75%	5.05%	5.20%	5.33%	5.46%	5.58%	5.67%
25%	4.63%	5.30%	5.39%	5.44%	5.53%	5.61%	5.68%	5.72%
50%	5.71%	5.81%	5.75%	5.74%	5.75%	5.76%	5.77%	5.77%
75%	6.85%	6.28%	6.13%	6.04%	5.98%	5.91%	5.86%	5.82%
90%	7.95%	6.80%	6.48%	6.33%	6.21%	6.05%	5.96%	5.87%
95%	8.56%	7.02%	6.68%	6.49%	6.32%	6.14%	6.01%	5.90%
99%	10.08%	7.53%	7.01%	6.79%	6.61%	6.30%	6.09%	5.96%
<i>Panel F: All Active Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	0.97%	4.64%	5.27%	5.51%	5.91%	6.36%	6.71%	6.89%
5%	2.90%	5.31%	5.83%	6.12%	6.42%	6.71%	6.90%	7.03%
10%	3.98%	5.78%	6.19%	6.33%	6.64%	6.86%	7.01%	7.12%
25%	5.59%	6.44%	6.75%	6.80%	6.98%	7.11%	7.19%	7.24%
50%	7.13%	7.27%	7.28%	7.34%	7.36%	7.37%	7.38%	7.37%
75%	8.84%	8.10%	7.94%	7.87%	7.74%	7.65%	7.57%	7.50%
90%	10.69%	9.03%	8.57%	8.35%	8.13%	7.89%	7.73%	7.63%
95%	11.80%	9.51%	8.94%	8.70%	8.35%	8.06%	7.84%	7.71%
99%	14.85%	10.90%	9.78%	9.20%	8.66%	8.31%	8.06%	7.80%
<i>Panel G: All Passive Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds		
1%	4.07%	5.90%	6.39%	6.48%	6.79%	7.08%		
5%	5.03%	6.28%	6.67%	6.80%	6.97%	7.18%		
10%	5.58%	6.57%	6.86%	6.97%	7.09%	7.23%		
25%	6.40%	6.97%	7.13%	7.19%	7.27%	7.33%		
50%	7.34%	7.44%	7.45%	7.45%	7.44%	7.44%		
75%	8.38%	7.94%	7.77%	7.71%	7.64%	7.55%		
90%	9.31%	8.42%	8.09%	7.92%	7.80%	7.65%		
95%	10.15%	8.72%	8.29%	8.07%	7.90%	7.70%		
99%	11.69%	9.07%	8.56%	8.33%	8.10%	7.81%		

**Table 10**

**Ending Wealth Per Dollar Invested by Quintile of  $R^2$  – Carhart Model Results**

This table displays select percentile values of terminal wealth in March 2013 on one dollar invested in December 1997 in portfolios that equally allocate capital to randomly selected funds that rank in a particular quintile of  $R^2$ . The portfolios are reconstituted every 12 months. Portfolios of 1, 5, 10, 15, 25, 50, 100, and 200 funds are examined. Panels A, B, C, D, and E pertain to funds in the bottom, 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, and top quintile of  $R^2$ . Panel F pertains to the entire universe of actively managed funds. Panel G pertains to the entire universe of index funds. The  $R^2$  estimation period is 24 months. The returns on the research factors and the risk-free rate are from Kenneth French’s website. The returns on mutual funds are from Morningstar Direct.

*Panel A: Quintile 1 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.47	\$1.70	\$2.04	\$2.29	\$2.42	\$2.64	\$2.80	\$2.95
5%	\$1.11	\$2.07	\$2.31	\$2.49	\$2.60	\$2.76	\$2.91	\$3.01
10%	\$1.40	\$2.24	\$2.45	\$2.60	\$2.71	\$2.84	\$2.95	\$3.04
25%	\$1.86	\$2.56	\$2.74	\$2.81	\$2.90	\$2.99	\$3.04	\$3.09
50%	\$2.68	\$3.02	\$3.08	\$3.09	\$3.11	\$3.14	\$3.15	\$3.15
75%	\$3.83	\$3.55	\$3.46	\$3.41	\$3.37	\$3.30	\$3.25	\$3.21
90%	\$5.16	\$4.17	\$3.88	\$3.70	\$3.61	\$3.47	\$3.35	\$3.27
95%	\$6.11	\$4.60	\$4.11	\$3.98	\$3.74	\$3.55	\$3.40	\$3.29
99%	\$8.48	\$5.47	\$4.71	\$4.41	\$4.02	\$3.75	\$3.53	\$3.34

*Panel B: Quintile 2 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.69	\$1.58	\$1.80	\$1.97	\$2.10	\$2.25	\$2.34	\$2.43
5%	\$1.16	\$1.79	\$1.99	\$2.11	\$2.21	\$2.32	\$2.40	\$2.46
10%	\$1.38	\$1.94	\$2.10	\$2.19	\$2.28	\$2.37	\$2.43	\$2.48
25%	\$1.78	\$2.20	\$2.29	\$2.33	\$2.39	\$2.45	\$2.48	\$2.51
50%	\$2.33	\$2.49	\$2.51	\$2.52	\$2.53	\$2.54	\$2.55	\$2.55
75%	\$3.03	\$2.84	\$2.77	\$2.71	\$2.67	\$2.64	\$2.60	\$2.58
90%	\$3.88	\$3.21	\$3.00	\$2.88	\$2.81	\$2.72	\$2.66	\$2.61
95%	\$4.45	\$3.41	\$3.15	\$3.00	\$2.89	\$2.78	\$2.70	\$2.63
99%	\$5.98	\$3.90	\$3.42	\$3.22	\$3.04	\$2.89	\$2.77	\$2.66

*Panel C: Quintile 3 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.88	\$1.56	\$1.74	\$1.86	\$1.99	\$2.10	\$2.17	\$2.26
5%	\$1.20	\$1.76	\$1.91	\$1.99	\$2.08	\$2.16	\$2.24	\$2.29
10%	\$1.38	\$1.86	\$2.01	\$2.07	\$2.13	\$2.20	\$2.26	\$2.31
25%	\$1.75	\$2.08	\$2.16	\$2.20	\$2.23	\$2.28	\$2.31	\$2.34
50%	\$2.26	\$2.34	\$2.36	\$2.34	\$2.36	\$2.37	\$2.37	\$2.37
75%	\$2.98	\$2.63	\$2.56	\$2.54	\$2.48	\$2.45	\$2.42	\$2.40
90%	\$3.76	\$2.93	\$2.76	\$2.69	\$2.61	\$2.53	\$2.46	\$2.42
95%	\$4.31	\$3.14	\$2.87	\$2.76	\$2.67	\$2.57	\$2.50	\$2.44
99%	\$5.58	\$3.46	\$3.04	\$2.96	\$2.78	\$2.65	\$2.53	\$2.47

(Continued)

**Table 10 - Continued**

<i>Panel D: Quintile 4 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.88	\$1.45	\$1.62	\$1.65	\$1.75	\$1.82	\$1.91	\$1.96
5%	\$1.14	\$1.58	\$1.70	\$1.79	\$1.83	\$1.89	\$1.94	\$1.99
10%	\$1.31	\$1.68	\$1.77	\$1.84	\$1.87	\$1.92	\$1.96	\$2.00
25%	\$1.57	\$1.84	\$1.89	\$1.93	\$1.95	\$1.97	\$1.99	\$2.02
50%	\$1.93	\$2.01	\$2.03	\$2.03	\$2.04	\$2.03	\$2.03	\$2.03
75%	\$2.41	\$2.24	\$2.19	\$2.15	\$2.13	\$2.09	\$2.07	\$2.06
90%	\$2.96	\$2.46	\$2.31	\$2.27	\$2.21	\$2.15	\$2.11	\$2.08
95%	\$3.36	\$2.60	\$2.40	\$2.31	\$2.24	\$2.18	\$2.13	\$2.09
99%	\$4.22	\$2.86	\$2.60	\$2.43	\$2.35	\$2.25	\$2.17	\$2.11
<i>Panel E: Quintile 5 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.89	\$1.37	\$1.50	\$1.59	\$1.64	\$1.70	\$1.74	\$1.79
5%	\$1.12	\$1.47	\$1.60	\$1.64	\$1.69	\$1.74	\$1.78	\$1.81
10%	\$1.25	\$1.56	\$1.65	\$1.69	\$1.72	\$1.76	\$1.79	\$1.82
25%	\$1.52	\$1.70	\$1.74	\$1.76	\$1.78	\$1.80	\$1.82	\$1.83
50%	\$1.79	\$1.83	\$1.83	\$1.84	\$1.84	\$1.85	\$1.84	\$1.84
75%	\$2.11	\$1.98	\$1.94	\$1.92	\$1.91	\$1.89	\$1.87	\$1.86
90%	\$2.41	\$2.11	\$2.04	\$2.01	\$1.97	\$1.93	\$1.90	\$1.87
95%	\$2.73	\$2.20	\$2.09	\$2.05	\$2.00	\$1.95	\$1.91	\$1.88
99%	\$3.23	\$2.42	\$2.21	\$2.14	\$2.08	\$2.00	\$1.94	\$1.90
<i>Panel F: All Active Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.72	\$1.53	\$1.72	\$1.79	\$1.90	\$2.04	\$2.14	\$2.20
5%	\$1.13	\$1.73	\$1.87	\$1.96	\$2.04	\$2.14	\$2.21	\$2.25
10%	\$1.32	\$1.85	\$1.96	\$2.03	\$2.11	\$2.19	\$2.24	\$2.28
25%	\$1.70	\$2.04	\$2.13	\$2.17	\$2.22	\$2.27	\$2.30	\$2.31
50%	\$2.12	\$2.29	\$2.32	\$2.35	\$2.35	\$2.36	\$2.36	\$2.36
75%	\$2.76	\$2.59	\$2.54	\$2.52	\$2.48	\$2.45	\$2.43	\$2.40
90%	\$3.57	\$2.93	\$2.79	\$2.70	\$2.62	\$2.54	\$2.48	\$2.45
95%	\$4.16	\$3.15	\$2.94	\$2.83	\$2.70	\$2.60	\$2.52	\$2.47
99%	\$5.97	\$3.84	\$3.31	\$3.06	\$2.83	\$2.70	\$2.60	\$2.51
<i>Panel G: All Passive Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds		
1%	\$1.40	\$1.92	\$2.06	\$2.09	\$2.18	\$2.28		
5%	\$1.66	\$2.03	\$2.14	\$2.19	\$2.25	\$2.31		
10%	\$1.79	\$2.11	\$2.21	\$2.24	\$2.28	\$2.33		
25%	\$2.03	\$2.23	\$2.29	\$2.31	\$2.34	\$2.36		
50%	\$2.33	\$2.40	\$2.40	\$2.40	\$2.40	\$2.40		
75%	\$2.70	\$2.57	\$2.51	\$2.49	\$2.47	\$2.44		
90%	\$3.06	\$2.75	\$2.63	\$2.57	\$2.53	\$2.47		
95%	\$3.41	\$2.85	\$2.70	\$2.62	\$2.56	\$2.49		
99%	\$3.99	\$3.00	\$2.82	\$2.71	\$2.63	\$2.53		

**Table 11**

**Ending Wealth Per Dollar Invested by Quintile of  $R^2$  – Cremers, Petajisto, and Zitzewitz Model Results**

This table displays select percentile values of terminal wealth in March 2013 on one dollar invested in December 1997 in portfolios that equally allocate capital to randomly selected funds that rank in a particular quintile of  $R^2$ . The portfolios are reconstituted every 12 months. Portfolios of 1, 5, 10, 15, 25, 50, 100, and 200 funds are examined. Panels A, B, C, D, and E pertain to funds in the bottom, 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, and top quintile of  $R^2$ . Panel F pertains to the entire universe of actively managed funds. Panel G pertains to the entire universe of index funds. The  $R^2$  estimation period is 24 months. The returns on mutual funds and the indices used in Cremers, Petajisto, and Zitzewitz’s model are from Morningstar Direct.

*Panel A: Quintile 1 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.66	\$1.70	\$2.15	\$2.20	\$2.40	\$2.60	\$2.78	\$2.90
5%	\$1.21	\$2.10	\$2.34	\$2.44	\$2.59	\$2.77	\$2.86	\$2.96
10%	\$1.49	\$2.26	\$2.47	\$2.56	\$2.69	\$2.84	\$2.92	\$2.99
25%	\$2.00	\$2.59	\$2.74	\$2.79	\$2.88	\$2.95	\$3.00	\$3.04
50%	\$2.89	\$3.05	\$3.09	\$3.09	\$3.08	\$3.09	\$3.09	\$3.10
75%	\$4.00	\$3.62	\$3.48	\$3.39	\$3.35	\$3.25	\$3.19	\$3.15
90%	\$5.39	\$4.15	\$3.84	\$3.70	\$3.53	\$3.40	\$3.30	\$3.20
95%	\$6.62	\$4.66	\$4.11	\$3.88	\$3.68	\$3.51	\$3.35	\$3.24
99%	\$10.13	\$5.88	\$4.64	\$4.26	\$4.04	\$3.64	\$3.48	\$3.31

*Panel B: Quintile 2 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.94	\$1.69	\$1.95	\$2.05	\$2.21	\$2.34	\$2.45	\$2.54
5%	\$1.26	\$1.89	\$2.11	\$2.22	\$2.31	\$2.42	\$2.50	\$2.57
10%	\$1.42	\$2.04	\$2.21	\$2.32	\$2.39	\$2.47	\$2.53	\$2.59
25%	\$1.83	\$2.28	\$2.42	\$2.47	\$2.51	\$2.54	\$2.58	\$2.62
50%	\$2.43	\$2.59	\$2.64	\$2.66	\$2.64	\$2.65	\$2.66	\$2.65
75%	\$3.14	\$2.95	\$2.88	\$2.84	\$2.81	\$2.76	\$2.72	\$2.69
90%	\$3.97	\$3.31	\$3.12	\$3.02	\$2.95	\$2.86	\$2.78	\$2.72
95%	\$4.57	\$3.50	\$3.27	\$3.16	\$3.04	\$2.90	\$2.81	\$2.75
99%	\$5.80	\$4.05	\$3.58	\$3.37	\$3.17	\$3.00	\$2.87	\$2.78

*Panel C: Quintile 3 Funds Portfolio*

Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.74	\$1.48	\$1.70	\$1.75	\$1.87	\$1.98	\$2.07	\$2.14
5%	\$1.08	\$1.63	\$1.81	\$1.90	\$1.97	\$2.06	\$2.12	\$2.17
10%	\$1.26	\$1.72	\$1.88	\$1.96	\$2.02	\$2.09	\$2.14	\$2.18
25%	\$1.59	\$1.92	\$2.04	\$2.07	\$2.11	\$2.16	\$2.18	\$2.21
50%	\$2.04	\$2.21	\$2.21	\$2.23	\$2.23	\$2.24	\$2.23	\$2.23
75%	\$2.64	\$2.47	\$2.41	\$2.38	\$2.34	\$2.31	\$2.28	\$2.26
90%	\$3.24	\$2.73	\$2.62	\$2.54	\$2.45	\$2.39	\$2.33	\$2.28
95%	\$3.73	\$2.90	\$2.73	\$2.61	\$2.53	\$2.44	\$2.35	\$2.30
99%	\$4.75	\$3.26	\$2.93	\$2.81	\$2.68	\$2.53	\$2.40	\$2.33

(Continued)

**Table 11 - Continued**

<i>Panel D: Quintile 4 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.84	\$1.42	\$1.57	\$1.64	\$1.73	\$1.83	\$1.88	\$1.96
5%	\$1.10	\$1.56	\$1.70	\$1.74	\$1.81	\$1.89	\$1.93	\$1.98
10%	\$1.23	\$1.66	\$1.76	\$1.81	\$1.85	\$1.91	\$1.95	\$1.99
25%	\$1.48	\$1.80	\$1.89	\$1.91	\$1.93	\$1.97	\$1.99	\$2.01
50%	\$1.91	\$2.00	\$2.03	\$2.02	\$2.04	\$2.03	\$2.03	\$2.03
75%	\$2.39	\$2.21	\$2.18	\$2.15	\$2.12	\$2.09	\$2.07	\$2.05
90%	\$2.89	\$2.46	\$2.31	\$2.26	\$2.20	\$2.15	\$2.11	\$2.07
95%	\$3.23	\$2.56	\$2.39	\$2.32	\$2.23	\$2.17	\$2.13	\$2.08
99%	\$4.21	\$2.78	\$2.55	\$2.44	\$2.32	\$2.23	\$2.15	\$2.11
<i>Panel E: Quintile 5 Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$1.03	\$1.45	\$1.58	\$1.62	\$1.70	\$1.77	\$1.81	\$1.86
5%	\$1.21	\$1.57	\$1.66	\$1.70	\$1.75	\$1.80	\$1.84	\$1.87
10%	\$1.34	\$1.63	\$1.71	\$1.75	\$1.78	\$1.82	\$1.85	\$1.88
25%	\$1.57	\$1.77	\$1.79	\$1.81	\$1.83	\$1.86	\$1.88	\$1.89
50%	\$1.84	\$1.90	\$1.89	\$1.89	\$1.90	\$1.90	\$1.90	\$1.91
75%	\$2.17	\$2.04	\$2.01	\$1.98	\$1.96	\$1.94	\$1.93	\$1.92
90%	\$2.54	\$2.19	\$2.11	\$2.07	\$2.03	\$1.99	\$1.96	\$1.93
95%	\$2.79	\$2.28	\$2.18	\$2.12	\$2.06	\$2.01	\$1.97	\$1.94
99%	\$3.37	\$2.44	\$2.27	\$2.20	\$2.16	\$2.06	\$2.00	\$1.96
<i>Panel F: All Active Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds	100 Funds	200 Funds
1%	\$0.72	\$1.53	\$1.72	\$1.79	\$1.90	\$2.04	\$2.14	\$2.20
5%	\$1.13	\$1.73	\$1.87	\$1.96	\$2.04	\$2.14	\$2.21	\$2.25
10%	\$1.32	\$1.85	\$1.96	\$2.03	\$2.11	\$2.19	\$2.24	\$2.28
25%	\$1.70	\$2.04	\$2.13	\$2.17	\$2.22	\$2.27	\$2.30	\$2.31
50%	\$2.12	\$2.29	\$2.32	\$2.35	\$2.35	\$2.36	\$2.36	\$2.36
75%	\$2.76	\$2.59	\$2.54	\$2.52	\$2.48	\$2.45	\$2.43	\$2.40
90%	\$3.57	\$2.93	\$2.79	\$2.70	\$2.62	\$2.54	\$2.48	\$2.45
95%	\$4.16	\$3.15	\$2.94	\$2.83	\$2.70	\$2.60	\$2.52	\$2.47
99%	\$5.97	\$3.84	\$3.31	\$3.06	\$2.83	\$2.70	\$2.60	\$2.51
<i>Panel G: All Passive Funds Portfolio</i>								
Percentile of Distribution	1 Fund	5 Funds	10 Funds	15 Funds	25 Funds	50 Funds		
1%	\$1.40	\$1.92	\$2.06	\$2.09	\$2.18	\$2.28		
5%	\$1.66	\$2.03	\$2.14	\$2.19	\$2.25	\$2.31		
10%	\$1.79	\$2.11	\$2.21	\$2.24	\$2.28	\$2.33		
25%	\$2.03	\$2.23	\$2.29	\$2.31	\$2.34	\$2.36		
50%	\$2.33	\$2.40	\$2.40	\$2.40	\$2.40	\$2.40		
75%	\$2.70	\$2.57	\$2.51	\$2.49	\$2.47	\$2.44		
90%	\$3.06	\$2.75	\$2.63	\$2.57	\$2.53	\$2.47		
95%	\$3.41	\$2.85	\$2.70	\$2.62	\$2.56	\$2.49		
99%	\$3.99	\$3.00	\$2.82	\$2.71	\$2.63	\$2.53		