Return Measures and Dollar Cost Averaging

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EXECUTIVE SUMMARY

While the debate concerning dollar cost averaging (DCA) versus lump sum (LS) investing rages on, we believe the debate should center on how to accurately measure investors' true returns. Understanding that the average person typically saves for retirement via periodic contributions to a retirement account, it is imperative to correctly measure both historical and expected investor performance. We believe a dollar-weighted return is superior to the traditional simple averages typically employed. During the period 1989-2008 an all equity fund earned a geometric average return of 8.43% while the realized dollar-weighted annual return earned by an investor making annual contributions to an all equity fund was only 5.41% due to the pattern of returns over the period. Our simulations suggest that such an extreme difference has a small probability of occurring. However, Dichev (2007) empirically demonstrates that by employing a geometric average return, researchers have overstated actual returns by as much as 5%. Further, if return distributions do not follow a normal distribution and/or the distributions are non-stationary, then the probability of such an outcome may be much higher. We show there is a decrease in ending wealth variability for a more balanced approach to investing. This implies that even young investors might consider a balanced asset allocation to reduce the likelihood of such a divergent outcome.

Return Measures and Dollar Cost Averaging

INTRODUCTION

Financial planners are routinely asked to make future value of annuity calculations regarding an investor's target wealth based on assumptions about the annual rate of return and an annual contribution amount. Planners, however, need to carefully examine the assumptions behind their computations. The fact that the computation assumes an *equal* annual rate of return, and not an *average* annual rate of return, is often overlooked. Furthermore, for investors who are making contributions to a retirement fund the *pattern* of returns over the planning horizon, not just the average return is what matters. In such situations, the geometric average annual rate of return is only relevant for an investor with a lump sum (LS) investment. The importance of return sequence for dollar cost averaging (DCA) versus LS investors is supported in the literature. Rozeff (1994) demonstrates that LS dominates DCA as long as there is a positive expected return. Likewise, Abeysekera and Rosenbloom (2000) show that LS is superior as long as stock returns are expected to exceed the risk-free rate. Otherwise, DCA may outperform a lump sum strategy. Dubil (2005) finds a similar result, but notes that the sequence of returns is key to determining which strategy will dominate. Atra and Mann (2001) also show that timing may be a key noting that the dominance of a particular strategy depends on the month of the year the strategy is started. Brennan, Li and Torous (2005) find support for dollar cost averaging in an environment where security prices display a pattern of mean reversion. This finding is consistent with an understanding that the return sequence is a key driver. Likewise, Greenhut (2005) demonstrates that DCA outperforms lump sum investing in a downward-trending market. As discussed below we argue that the focus should be on effectively gauging the true return investors have earned (or can expect to earn). As demonstrated by Dichev (2007), we (as planners) are arguably overstating true returns when we employ a traditional geometric average.

Young investors are often urged to use high allocations to equities in their retirement accounts during their accumulation years because of the higher long-run returns on equities. These investors have a relatively long time horizon over which to bear the risk of equities, and the fact that the investor is "dollar cost averaging" into equities by making periodic contributions. The dollar cost averaging, it is popularly alleged, ensures than investors "purchase more shares when the stock market goes down, and purchase fewer shares when the stock market goes up," which is supposedly beneficial. However, dollar cost averaging can result in lower returns if the investor faces high returns in the early periods followed by low returns in the later periods during the wealth accumulation phase.

The period 1989-2008 provides an interesting case study of how the realized return earned by an investor on his or her contributions can diverge from the geometric average return of the underlying portfolio due to the sequence of the returns. The realized annual rate of return earned by an investor making equal annual contributions to a 100% equity portfolio was only 5.41% while the portfolio itself earned an 8.43% geometric average return over the same period. This divergence is because of the dramatically different geometric means during the two decades: 19.21% during 1989-1998 and -1.38% during 1999-2008. Financial planners need to consider this possible difference in the realized effective annual return versus the realized geometric mean, in addition to the possible variation in the geometric mean itself, on the variability of ending wealth. As Dichev (2007) argues, "buy-and-hold returns are essentially *security* returns, while *investors'* returns are determined not only by the returns on the securities they hold but also by the timing and the magnitude of their capital flows into and out of these securities." Dichev's study explicitly recognizes this difference in return definition and develops a new and more accurate measure of stock investors' actual returns. This return measure is the traditional dollar-weighted return, often referred to as the internal rate of return (IRR) in corporate finance. It specifically accounts for, not only security returns, but also the timing of investor cash flows. We believe this is a superior measure when gauging the average client's progress toward retirement.

The simple example given by Dichev eloquently demonstrates the clear difference between a security's return versus the investor's return. Following Dichev, assume an investor buys 100 shares of ABC stock at the beginning of period 1 for $10 and an additional 100 shares at the end of period 1 for $20 then the stock price settles at $10 per share at the end of period 2. The simple geometric average return for the security is:

$$R_{ABC} = \lfloor (1+100\%)(1-50\%) \rfloor^{1/2} - 1 = 0,$$

And the true return to the investor is:

CF_0 = -$1,000; CF_1 = -$2,000; CF_3 = +$2,000

Solve for the Internal Rate of Return (IRR) = -26.8%.

There is a clear difference in the “true” return for the client after correctly accounting for both the security’s return and the cash flows from the perspective of the investor. Accordingly, we are erroneously interpreting the data when we examine, or input as an assumption, solely the return on the security with no regard for the impact of the client’s capital inflows and outflows.

Our paper proceeds as follows: First we examine realized returns from the period 1989-2008 from the perspective of both lump sum investors and investors who are making annual contributions to illustrate the difference in outcomes for the realized annual returns on contributions depending on the order of returns. Then we perform Monte Carlo simulations using parameter estimates from 1989-2008 to gauge the probability of such an outcome of more than a 300 basis point difference between the realized geometric mean and the realized effective annual return on contributions. We then perform the analysis on various asset allocation mixes to gauge the benefit of using a balanced asset mix instead of 100% equities during the accumulation phase. Finally, we highlight some key findings and the implications for individual investors and their advisors.

DATA AND METHODOLOGY

The data for this study were taken from the Ibbotson© SBBI© 2009 Classic Yearbook. We focus on three fundamental asset classes: equities, bonds, and cash. For equities, we use the SBBI Large Company Stocks total return series. For bonds, we use the Intermediate-Term Government Bonds total return series. Finally, for cash, we use the U.S. Treasury Bills total return series. The annual returns of each asset class are examined for the 20-year period 1989 – 2008. A summary of the returns are shown in Table 1. The actual annual returns are shown in Appendix A.

Table 1
Total Returns 1989 - 2008

	Large Company Stocks	Intermediate-Term Government Bonds	U.S. Treasury Bills
Geometric Average	8.43%	7.48%	4.25%
Arithmetic Average	10.36%	7.65%	4.27%
Standard Deviation	19.99%	6.02%	2.01%

Figure 1 shows the change in value of $1 deposited at the beginning of 1989. Given the relatively large standard deviation, it is not surprising that the value of the 100% stock portfolio varies dramatically over the 20-year period examined. The growth in the 100% bond and 100% cash portfolios are much smoother producing a lower variation.

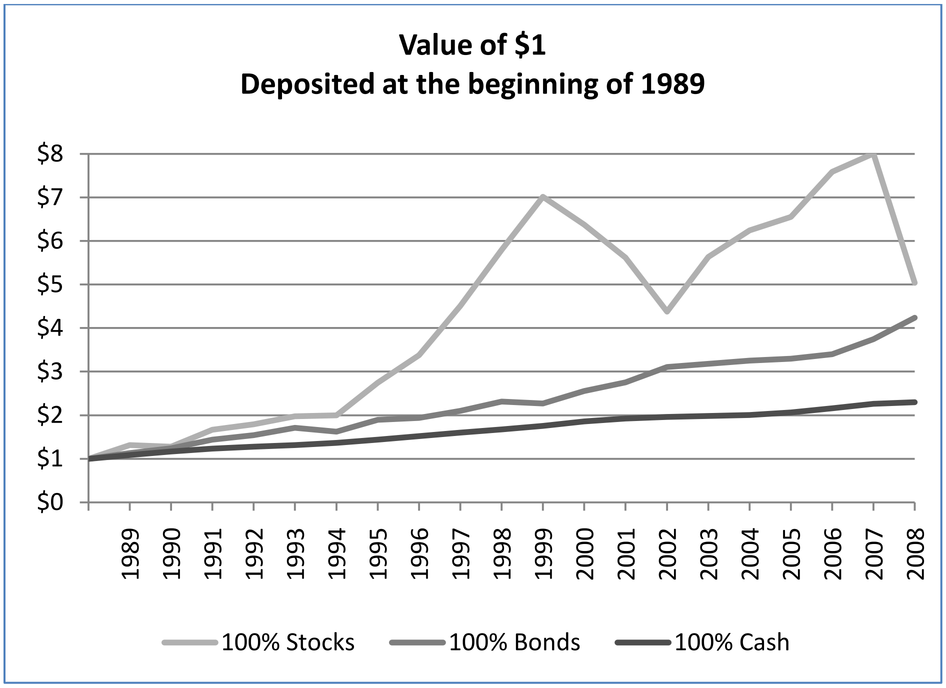

Figure 1 – Growth in a $1 Deposit for the Period 1989-2008

When projecting future portfolio values, the historical geometric average is often used. Figure 2 shows the annual growth in the value of $1 using the actual annual returns (IRRs) for a 100% stock portfolio from 1989 - 2008. It also shows how that $1 would have grown if the order of the returns had been reversed; 2008's return occurred in the first year, 2007's return occurred in the second year, etcetera. Although the two paths are quite different, the value of the $1 deposited at the beginning of 1989 grows to the same $5.04 under either return sequence. The geometric average return under either return sequence is 8.43%. Thus, using the geometric average return in forecasting terminal portfolio values is justifiable for a lump-sum investment. The order of the annual returns does not impact the terminal wealth.

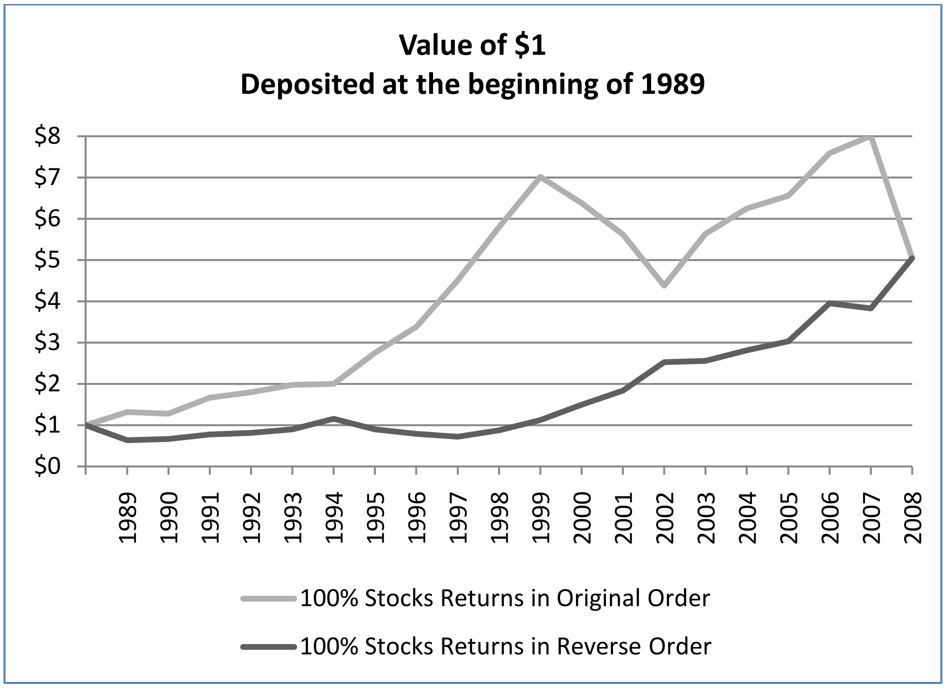

Figure 2 – Growth of a $1 Investment with Different Return Sequencing

However, using a constant annual return ignores the potential variation in returns over the time period. Although this does not impact the forecast for the terminal value of a deposited lump-sum invested, it can lead to misleading forecasts when periodic deposits are made. Figure 3 shows the change in the value of a portfolio where $1 is invested in 100% stocks at the beginning of each year for 20 years. Although the annual returns are identical to those in Figure 2, the terminal wealth values in Figure 3 are quite different for the two return sequences. When periodic investments are made, the order of the returns is extremely important. Using the actual annual returns, the series of $1 investments grows to $36.39, while when the reverse annual returns are assumed the same investments grow to $88.92, an increase of 144%. This dramatic difference clearly demonstrates the need for a more accurate measure of returns when the client is making period contributions. The dollar-weighted return measure is superior because it correctly gauges the actual performance of the client.

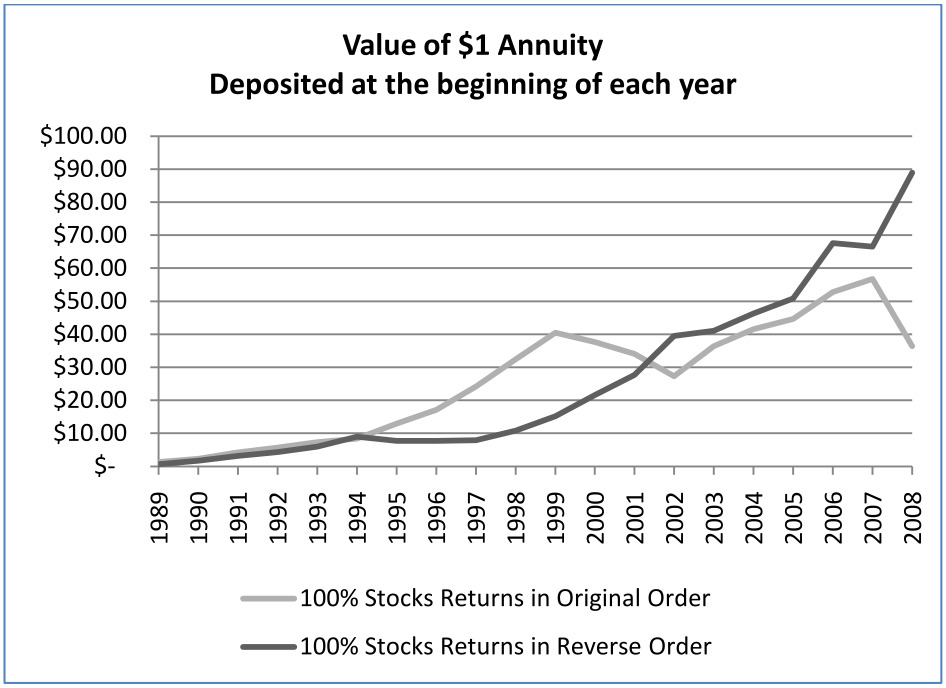

Figure 3 – Growth of a $1 Annuity with Different Return Sequencing

Table 2 and Figure 4 show the effective annual rate and the terminal wealth for the different asset classes based on the annual returns in the original order and then in the reverse order. The effective annual rate (EAR) is the rate that equates the terminal wealth with future value of the series of $1 investments. Here again, the EAR is calculated consistent with the IRR method discussed earlier. For the intermediate Treasury bonds and the Treasury bills, the difference between the EAR for the original and reverse order returns is less than 1%. However, the difference is over 7% for the stock portfolio. The relatively large standard deviation for stocks is the driver of this difference.

Table 2
Effective Annual Returns 1989 - 2008

	Large Company Stocks	Intermediate-Term Government Bonds	U.S. Treasury Bills
Terminal Value Original Order	$36.39	$43.15	$29.82
EAR Original Order	5.41%	6.86%	3.66%
Terminal Value Reverse Order	$88.92	$48.74	$33.93
EAR Reverse Order	12.77%	7.88%	4.80%

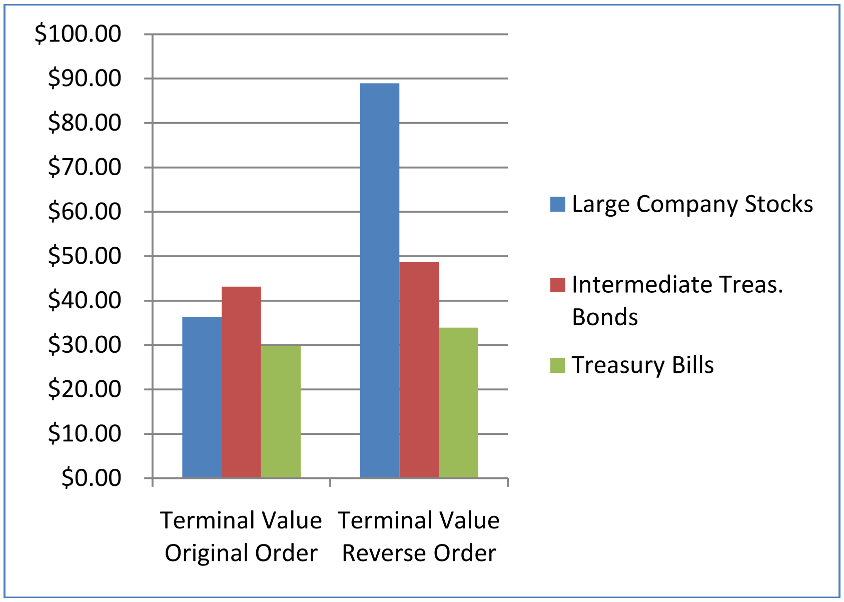

Figure 4 – Terminal Wealth for Asset Classes with Different Return Sequencing

To more closely examine the impact of the order of returns on terminal wealth, the Crystal Ball statistical package is used to conduct Monte Carlo simulations. The simulation examines the variations in returns and terminal wealth for a twenty-year period. For each year, returns are drawn from a normal distribution using the actual arithmetic average and standard deviation for the period 1989 – 2008, as shown in Table 1. For example, when simulating the 100% stock portfolio, twenty annual returns are drawn from a normal distribution with a mean of 10.36% and a standard deviation of 19.99%. We examine portfolios of 100% stocks, 100% bonds, 100% bills, 50% stocks & 50% bonds, 50% stocks & 50% bills, 50% bonds & 50% bills, and 33.3% stocks, 33.3% bonds, and 33.3% bills. For each portfolio, 500,000 simulations of 20 years each are conducted.

Table 3 shows the values for various portfolios assuming $12,000 per year was deposited at the beginning of each year beginning in 1989 and ending in 2008. The Actual row contains the terminal portfolio value assuming the actual asset class returns were earned. The Mean, Median, Minimum and Maximum rows contain the values of the mean, median, minimum, and maximum terminal portfolio values using the 500,000 Monte Carlo simulations assuming the average annual return and standard deviation for each of the asset classes during the 1989 – 2008 period. The Percentile rows list the terminal portfolio values using the Monte Carlo simulations for the various percentiles.

Table 3
Portfolio Ending Wealth

	100% Cash	100% Bonds	100% Stock	50% Stock/ 50% Cash	50% Stock/ 50% Bonds	50% Bonds/ 50% Cash	1/3 Stock/ 1/3 Bonds/ 1/3 Cash
Actual	$357,818.56	$517,822.73	$436,631.13	$412,827.56	$513,651.75	$431,641.10	$455,766.11
Mean	$383,043.99	$568,668.98	$789,082.83	$546,435.13	$669,649.67	$465,600.22	$553,728.52
Median	$382,403.12	$559,875.52	$659,480.91	$521,466.49	$639,418.85	$463,361.09	$541,590.44
Minimum	$289,377.78	$261,437.22	$48,012.86	$138,954.20	$164,203.41	$299,291.09	$216,874.72
Maximum	$498,830.88	$1,227,985.10	$10,814,541.22	$2,571,605.72	$2,666,099.12	$756,225.22	$1,468,966.30
Percentiles							
0%	$289,377.78	$261,437.22	$48,012.86	$138,954.20	$164,203.41	$299,291.09	$216,874.72
10%	$355,683.97	$447,565.80	$310,392.61	$354,612.58	$435,101.16	$408,329.77	$414,781.24
20%	$364,642.68	$483,339.15	$400,983.55	$404,310.48	$496,060.80	$426,330.45	$454,381.28
30%	$371,208.30	$510,745.14	$483,543.93	$444,998.90	$545,666.63	$439,908.54	$485,337.26
40%	$376,966.98	$535,577.36	$567,738.57	$482,776.13	$592,415.80	$451,885.39	$513,612.80
50%	$382,403.07	$559,875.12	$659,480.45	$521,466.25	$639,418.66	$463,360.79	$541,590.32
60%	$387,910.24	$585,292.09	$766,914.24	$562,723.82	$689,989.88	$475,057.18	$570,846.11
70%	$393,948.44	$613,781.09	$901,369.14	$611,388.88	$749,226.33	$487,922.21	$604,253.96
80%	$401,121.06	$648,855.88	$1,089,123.51	$673,339.52	$824,807.85	$503,396.45	$645,908.20
90%	$411,186.20	$700,870.92	$1,417,866.74	$769,293.95	$941,870.38	$525,869.26	$707,844.90
100%	$498,830.88	$1,227,985.10	$10,814,541.22	$2,571,605.72	$2,666,099.12	$756,225.22	$1,468,966.30

One of the most striking results is the difference between the mean and median values. In every case, the mean portfolio value is more than the median portfolio value. This result is driven by the log-normal nature of the distribution of portfolio values, as shown in Figure 5. The most dramatic case can be seen with the 100% stock portfolio. The portfolio value at the 90th percentile is approximately $1.4 million while the maximum portfolio value is approximately $10.8 million. This skewed distribution makes it misleading to use the mean portfolio value. The median value would be a more realistic estimate of the portfolio's terminal value.

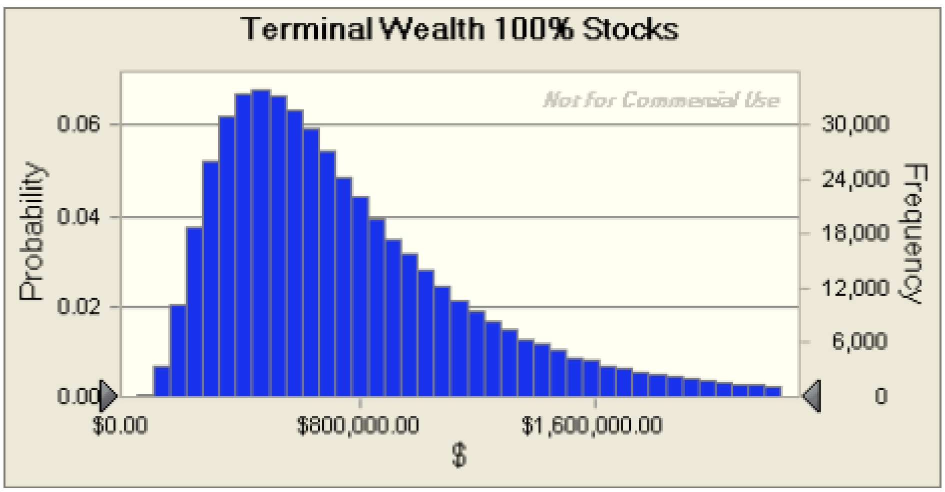

Figure 5 – Distribution of Terminal Wealth from Simulations

Another surprising result displayed in Table 3 is that there is very little difference in the median terminal wealth for the 100% stock portfolio and the 50% stocks/50% bonds portfolio. However, the 50%/50% portfolio has significantly less variation. An investor would only be significantly better off in the 100% stock portfolio approximately 35% of the time. This would suggest that investors are better off with a more balanced approach to investing, even during the accumulation phase of their investing timeframe.

We also look at the probability of observing a 300 basis point difference between the effective annual return that an investor with regular periodic contributions would earn and the geometric average return, as was the case during the 1989 – 2008 timeframe. Given the actual return sequence observed, this divergence resulted in the 100% stock portfolio ending up with a lower terminal value than the 100% Bond portfolio even though stocks earned a higher geometric mean over the period. Figure 6 plots the difference between the effective annual return and the geometric average return for the 500,000 Monte Carlo simulations using the actual arithmetic mean (10.36%) and standard deviation (19.99%). Table 4 lists the percentiles for the differences between the effective annual return and the geometric average return. The data indicate that the expected difference between the effective annual return and the geometric average return is 16 basis points. If we look at both tails of the distribution, there is approximately a 40 percent chance the investor will realize an actual return more than 200 basis points different than the expected return using the geometric average. Significantly more than a 300 basis point difference is expected approximately 20 percent of the time.

A lower-tail shortfall of 300 basis points, as experienced in the actual data, has less than a 10% probability of occurring. It must be pointed out that the Monte Carlo simulation assumes that the returns are drawn from a stationary normal distribution. If the return generating process does not fit this assumption then the probability of a large divergence in realized return versus geometric mean may be higher than the simulation suggests. Therefore, this potential divergence should be of concern to financial planners even though the expected divergence may be small. It appears that a more balanced asset allocation would reduce this potential divergence.

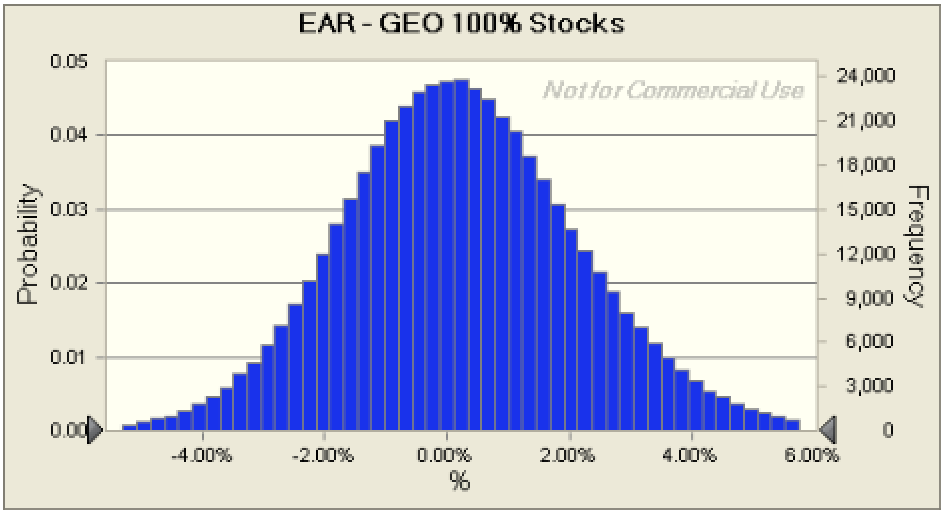

Figure 6 – Distribution of EAR minus Geometric Average Returns

Table 4	
Percentiles	Forecasted EAR - GEO
0%	-11.45%
10%	-2.20%
20%	-1.40%
30%	-0.82%
40%	-0.32%
50%	0.16%
60%	0.64%
70%	1.17%
80%	1.80%
90%	2.74%
100%	13.30%

Table 4 – Percentiles for Ear minus Geometric Average Returns

One further point to note is that the proportion of wealth accumulated over time is not as evenly distributed as one may think. Figure 7 graphs the proportion of ending "target" wealth accumulated as time passes. The figure assumes a 40 year horizon with equal annual contributions and a constant annual rate of return. As noted in the figure, approximately 50 percent of the client's targeted ending wealth comes in the final eight years of savings. This illustration further highlights the potential need for a more balanced approach throughout the investor's lifecycle.

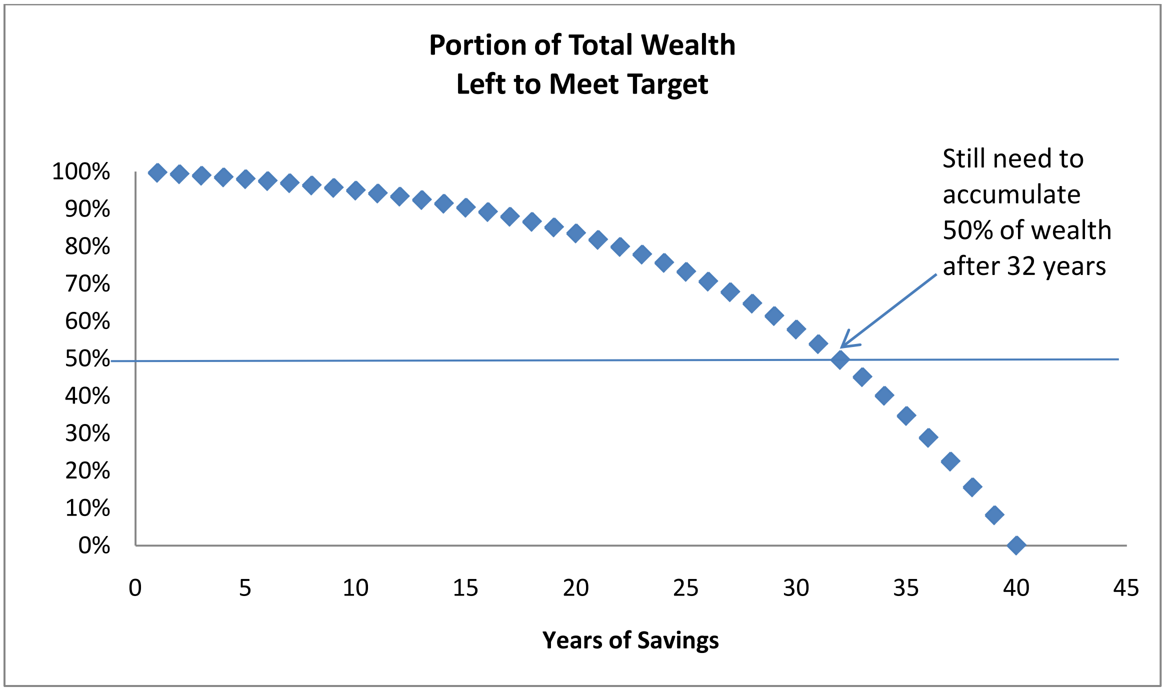

Figure 7 – Proportion of Wealth Accumulation over Time

SUMMARY AND CONCLUSION

Since the actual return realized by an investor making annual contributions to a portfolio can differ dramatically from the geometric mean earned by the underlying portfolio, financial planners need to take this potential difference into account when advising clients about the projected level and variability of ending wealth of various investment strategies. Following Dichev (2007) we refocus the dollar-cost averaging versus lump sum debate on this issue. For investors making contributions to retirement accounts the dollar-weighted average return is the relevant return measure, not the geometric mean. During the period 1989-2008 an investor making equal annual contributions to a 100% equity portfolio earned a realized effective annual return (as measured by the dollar-weighted average return) of only 5.41% while the portfolio itself earned a geometric mean of 8.43%. In fact, the investor would have ended with less ending wealth in a 100% stock portfolio than with a 100% bond portfolio even though stocks earned a higher geometric average return (8.43% versus 7.48%) over the period.

The sequence of returns can have a dramatic effect on realized returns to an investor making contributions to a retirement fund. Over the period studied if the returns had occurred in reverse order the realized return on the 100% stock portfolio would have been 12.77% instead of 5.41%. In simulations, the distribution of ending wealth in a 100% stock portfolio is dramatically skewed implying that planners should focus on the expected median ending wealth and not the average ending wealth when advising clients about possible outcomes. Further, simulations suggest that large differences (200 basis points or more) between geometric mean and realized returns to an investor making annual contributions have relatively high probabilities of occurring, adding to the variability of ending wealth. While the negative 300 basis point difference observed during 1989-2008 has a probability of less than 10% in the simulation, if the return distribution is not normal and/or is non-stationary, then the probability of such a divergent outcome may be much higher. Finally, comparisons of balanced asset allocations suggest that the variability of realized return versus geometric return can be reduced by adding diversification across asset classes. This implies that even young investors should consider a balanced asset allocation, and since much of the effect on ending wealth is driven by returns in the latter few years, it further reinforces the benefit of maintaining a balanced allocation throughout the accumulation phase.

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APPENDIX A

Average Annual Returns

Year	Large Company Stocks	Intermediate Treas. Bonds	Treasury Bills
1989	31.69%	13.29%	8.37%
1990	-3.10%	9.73%	7.81%
1991	30.47%	15.46%	5.60%
1992	7.62%	7.19%	3.51%
1993	10.08%	11.24%	2.90%
1994	1.32%	-5.14%	3.90%
1995	37.58%	16.80%	5.60%
1996	22.96%	2.10%	5.21%
1997	33.36%	8.38%	5.26%
1998	28.58%	10.21%	4.86%
1999	21.04%	-1.77%	4.68%
2000	-9.10%	12.59%	5.89%
2001	-11.89%	7.62%	3.83%
2002	-22.10%	12.93%	1.65%
2003	28.68%	2.40%	1.02%
2004	10.88%	2.25%	1.20%
2005	4.91%	1.36%	2.98%
2006	15.79%	3.14%	4.80%
2007	5.49%	10.05%	4.66%
2008	-37.00%	13.11%	1.60%