Optimal Tax-efficient Planning of Withdrawals from Retirement Accounts

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Abstract

Retirees must decide when and how much to withdraw from their retirement accounts. These decisions are complicated by tax laws and other factors. We develop a new approach to planning retirement withdrawals that satisfies living expense goals and uses mathematical optimization to prescribe how much to withdraw from both taxable and tax-deferred savings while dealing directly with complicating factors. The resulting withdrawal plan is tax-efficient since final accumulated wealth is maximized by controlling the impact of federal taxes. We demonstrate the potential for increasing retirees' wealth through typical scenarios that compare our tax-efficient results with those determined through rules of conventional wisdom.

1. Introduction

As the more than 77 million baby boomers reach retirement age, retirees, financial planners and academics increasingly focus on problems related to saving and planning for retirement. Sources of retirement income include Social Security, taxable savings and a wide range of tax-deferred retirement plans, e.g., pensions, 401(k) plans, IRAs, and annuities. Retirees must decide when and how much to withdraw from each wealth source to supplement Social Security in order to pay living expenses, pay taxes and satisfy federal required minimum distributions (RMDs) for tax-deferred plans. The withdrawal planning process is complicated by tax laws and possibly by employer retirement plan restrictions on how money must be withdrawn. Often, financial institutions and wealth mangers deal with these complexities by using rules of "conventional wisdom" (CW) and average tax rates. This process is not always straightforward (Ruffenach 2005, p. R4):

> "The other piece of the puzzle is tax efficiency. You're probably familiar with the conventional wisdom: Draw down your taxable accounts first; then turn to tax-deferred accounts, like IRAs… In this way, tax-deferred assets get more time to grow. But the sequence isn't always that simple"

To meet the challenges of planning retirement withdrawals, we describe a new approach that can be used by retirees and their wealth managers after specifying living expense goals. Living expenses during retirement are readily estimated by applying anticipated cost-of-living adjustments to past bills and then adding or subtracting amounts to accounts to achieve desired expectations for future quality of life. Our withdrawal planning approach includes a mathematical optimization model that determines how much to withdraw from each wealth

source for each year of a fixed planning horizon to achieve the following retirement financial goals:

- Satisfy specified living expenses.
- Meet requirements imposed by federal laws and retirement plans.
- Maximize final accumulated wealth.

A good approximation of federal income taxes is determined as an integral part of the process of meeting the above goals. Since final accumulated wealth is maximized by controlling the impact of taxes, the resulting withdrawal plan is considered tax-efficient.

Through examples, we demonstrate the benefits of our tax-efficient approach over a commonly used CW rule to determine retirement withdrawals: draw down taxable savings first, then tax-deferred savings. Other CW rules often include:

- Initially, withdraw a sustainable percent (e.g., 4% or 5%) of one's nest-egg to supplement Social Security; then adjust this amount in subsequent years for inflation. "Sustainable" refers to the probability of not exhausting one's wealth over a given time horizon (say, 30 years). This probability is assessed using Monte Carlo simulation.
- Determine living expenses by deducting an average federal income tax percent (e.g., 13%, 22%, etc.) from total withdrawals.

While CW rules provide useful guidance to wealth managers, our tax-efficient planning approach offers the following advantages over these rule-based methods:

- Retirees can better relate to withdrawal plans that meet specified needs, rather than plans based on withdrawal percents. Firstly, sustainability is not an easily understood concept by

most retirees. Secondly, percentage-based withdrawal plans frequently provide less income than needed early in retirement when retirees are active and/or may specify more income than needed later in retirement when retirees travel less, mortgages are paid off, etc.

- For the same living expenses, the use of mathematical optimization guarantees that final accumulated wealth is higher (often substantially higher) than that resulting from the use of CW rules.
- Due to RMDs and possibly other restrictions on retirement withdrawals, attempting to revise an initial CW rule-based plan to gain tax efficiency is tedious at best – even using a well structured spreadsheet that accounts for many key features of the tax laws. Our tax-efficient withdrawal plan for a 25-year planning horizon is determined using Excel based software in less than one minute. This speed provides a wealth manager with enormous potential for quickly analyzing alternative scenarios. For example, what is the impact on the retiree's final accumulated wealth if living expenses are increased or decreased? Other examples include evaluating the impact of alternative investments, and, whether or not to retire earlier (or later).

While our approach does not directly consider the impact of market volatility on savings accumulations, annual plan updates should suggest the need for appropriate adjustments to expenses and/or the distribution of equities and fixed income to achieve amounts of accumulated wealth that satisfy acceptable levels of perceived risk. Thus, we feel our approach discussed in this paper should be of great value to wealth managers in allowing them to quickly determine a realistic tax-efficient withdrawal plan that achieves an acceptable quality of life and a comfortable level of risk for the retiree.

2. Literature review

Clements (2005 – April), Ruffenach (2005) and Whitaker (2005) discuss problems associated with retirement income planning and the need for better ways to plan withdrawals. The importance of considering taxes when planning retirement withdrawals is emphasized by Clements (2005-February) and Whitaker (2005) but no general tax-efficient approach is suggested. The complexity of the tax laws result in contrasting recommendations on how to save for retirement income. For example, Darlin (2007) argues for saving less and Hube (2007) suggests taking advantage of capital gains for taxable savings rather than tax-deferred plans.

Early academic research on retirement withdrawals follow the lead of Bengen (1994) who studies withdrawal rates (i.e., the annual percent of initial retirement wealth) that provide sustainable lifetime income. Pye (2000) and Cooley, Hubbard and Walz (2003) use simulation using Monte Carlo and/or "overlapping periods" to show that a 4% to 5% initial withdrawal rate is highly sustainable for commonly assumed portfolio ROR distributions. Sacks (2004), Guyton and Klinger (2006) and Hubbard (2006) extend earlier simulation studies by evaluating elaborate rules (i.e., heuristics) that adjust withdrawal rates for factors such as inflation and a portfolio's prior annual ROR. Spitzer and Singh (2006) and Horan (2006) consider a wider range of account types while Stout and Mitchell (2006) analyze the additional dimension of life expectancy. Van Harlow and Feinschreiber (2006) provide examples of how heuristic rules and "conventional wisdom" can be far from "optimal" for many practical situations. The importance of specifying expenses and how they might change over time is discussed by Nersesian (2006), Levitz (2006) and Whitehouse (2006). Robinson (2007) recommends an approach to planning

withdrawals that meet living expense needs. The reduction of risk resulting from regular adjustments of the distribution of equities and fixed income is discussed by Fullmer (2007).

Ragsdale, Seila and Little (1994) use linear programming to determine tax-efficient optimal withdrawal plans from tax-deferred savings given total income needs (including taxes). While this approach provides useful insights, the design and scope of the model are limited with respect to problems currently faced by retirees (for example, taxable savings is not considered). Furthermore, the use of the optimization model as part of the larger process of wealth management is not comprehensively addressed.

3. The optimization model

Elements of the optimization model include a defined measure of wealth, income sources, model variables, model constraints and a solution algorithm. A prototype mathematical model that is used for the specific scenarios in Section 4 is included in the Appendix. It is important to emphasize that this prototype is just an example; the current state of the art in mathematical programming modeling and software can accommodate most features of current tax laws (e.g., capital gains distributions) as well as other income sources (e.g., Roth IRAs) and transfers of funds from one account type to another.

3.1 Defined measure of wealth

Our prototype model maximizes accumulated wealth at the end of the planning horizon. Other measures could be considered. For example, one that might favorably impact estate beneficiaries is accumulated taxable savings at the end of the planning horizon.

3.2 Income sources

The retiree must specify approximate income from sources such as Social Security, pensions, taxed earnings, inheritances, etc. Our model determines income from two accounts: taxable savings and tax-deferred savings (e.g., traditional IRAs, 401(k) plans, etc.); other sources (Roth's, tax-free municipal bonds, traditional annuities, etc.) could be included in future models. The retiree must specify initial amounts for each account.

3.3 Model variables (i.e., values that are determined by the model for each year)

a. Amounts to withdraw from each account to cover living expenses, federal taxes and federal RMDs for tax-deferred accounts.
b. Amount to be transferred from tax-deferred accounts to taxable savings. One use for these variables is to optimally balance tax impact over the planning horizon.
c. Other auxiliary variables needed to determine federal taxes.

3.4 Model constraints

a. A withdrawal from an account must be no greater than the amount available.
b. Living expenses must be met.
c. Federal RMDs must be satisfied.

The model also includes auxiliary variables and associated constraints that are needed to compute approximate federal taxes.

3.5 Solution algorithm

Due to the complex nature of the tax laws, solving a general model covering all aspects of federal taxes would require use of a constrained non-linear optimization algorithm. These algorithms are generally not time-efficient and would greatly expand the time needed for scenario analysis. However, time-efficient mixed integer linear programming models can be

designed that adequately account for many of the complexities of the tax laws – see, for example, Ragsdale, Seila and Little (1994) and Hadley (1962).

4. Results for typical scenarios

This section includes scenarios that demonstrate some specific benefits of our proposed retirement account withdrawal planning approach and prototype optimization model.

4.1 Data

We classify data into two types:

a. Not subject to change (fixed): We include planning horizon, retiree age, RMD coefficients, Social Security payments, tax exemptions, tax brackets and tax rates; where appropriate, cost of living adjustments are applied for each year during the planning horizon.
b. Planning related (discretionary): For each year during the planning horizon we include estimated interest rate or ROR for each type of account, specified living expenses and a sub-total of tax-deductible expenses.

Data for the scenarios are based on examples in the literature, the authors' personal experience, professional experience (one author, Sumutka, is a practicing CPA) and consultation with wealth managers. We consider a couple, ages 65 (husband) and 63 (wife) who are planning 25 years of retirement with a total current wealth (i.e., retirement portfolio) of $1,000,000. This is divided between two accounts:

1. Taxable savings investments: $100,000 – average ROR of 5.5%.
2. Tax-deferred savings: $900,000 – average ROR of 7.5%.

The husband's annual Social Security starts at $16,000 and the wife adds $8,487 in year three (this is one-half of the husband's amount in year three) of the planning horizon. RMDs begin in year 5 when the husband reaches 70½ . Tax brackets and tax exemptions are initialized at 2009

rates. Annual cost-of-living adjustments are applied at an annual rate of 3% to Social Security, tax brackets and tax exemptions.

4.2 Assumptions

All results are based on the following assumptions:

a. Withdrawals from an account are proportional to the internal asset allocation of that account. For example, if the tax-deferred account is 40% bonds/60% equities, then a $1000 withdrawal will be $400 from bonds and $600 from equities.
b. Social Security is taxable as per the worksheet in the instructions for IRS form 1040.
c. Funds needed to meet living expenses are withdrawn at the beginning of the year. Withdrawals from the tax-deferred account that exceed the amount needed to meet specified living expenses (e.g., due to RMDs) are withdrawn at the end of the year and deposited in the taxable savings account.

4.3 Living expenses

Living expenses are specified by the couple in a budget worksheet. Subtotals for the current year (at husband age 65) are shown in Table 1.

[Insert Table 1 here]

Federal taxes are not included since the model determines approximate annual amounts. Mortgage payments are included in the Household category. Current itemized deductions are $20,800. Projected expenses for each year are listed in Tables 2 and 3 under Specified Living Expenses. These reflect increases of about 3% per year, except for a reduction after age 80 when the mortgage payments of $15,000/year cease.

4.4 CW rule-based withdrawal plan

The data in Section 4.1 and assumptions in Section 4.2 are sufficient to determine a CW plan based on the rule of withdrawing from taxable savings first before withdrawing tax-deferred retirement savings. To allow for a proper comparison with the tax-efficient plan, living expenses are maintained at identical levels. Total withdrawals for the two withdrawal plans differ, however, due to differences in federal taxes. Our use of specified living expenses differs from the common practice of specifying an initial withdrawal rate and increasing annual total requirements by a cost of living adjustment. When this is done, after-tax living expenses are determined after subtracting an allowance for federal taxes; therefore, living expenses would necessarily be different for the two comparative plans.

Table 2 contains results for the CW rule-based plan.

[Insert Table 2 here]

All taxable savings are used at ages 66 and 67. Starting at age 67 deferred savings are needed to meet planned cash flow until age 80. RMDs begin at age 70 but withdrawals from tax-deferred savings needed to meet desired living expenses exceed the RMDs until age 81. Beginning at age 81 the RMD amount exceeds the amount needed for living expenses. The excess funds are transferred to taxable savings at year end and are used to satisfy future expenses. At age 86, taxable savings have accumulated from excess RMDs to the extent that they are sufficient to meet living expenses; all of the RMD is therefore deposited at year end to taxable savings. Final accumulated wealth in both savings accounts is $1,537,400.

4.5 Optimal tax-efficient withdrawal plan.

Optimization software used to run the model with Microsoft Office's Excel is the Premium Solver Platform supplied by Frontline Systems and described in Fylstra, Lasdon, Watson and

Waren (1998). Results for the optimal tax-efficient withdrawal plan are presented in Table 3 and shown graphically in Figure 1.

[Insert Table 3 and Figure 1 here]

A mix of taxable and deferred savings is withdrawn through age 74. The inclusion of deferred savings during this period is offset by tax deductions and exemptions that range from $27,000 to $43,000 (not included in Table 2) over the 25 year horizon. Withdrawal of tax-deferred funds prior to age 74 optimally fills lower tax brackets – note that little income tax is paid from ages 66 to 70 and from ages 72 to 74. After age 81, tax-deferred savings withdrawals exceed the amount needed to meet total cash-flow with the excess transferred to taxable savings at year end. As with the CW plan, at age 86, taxable savings have accumulated from excess RMDs to the extent that they are sufficient to meet living expenses; all of the RMD is therefore deposited at year end to taxable savings. Final accumulated wealth in both savings accounts is $1,591,700 – an increase of 3.5% over the final accumulated CW plan wealth.

4.5 Wealth planning implications of the tax-efficient withdrawal plan.

Accumulated final wealth and withdrawal levels provided by the tax-efficient plan may suggest alternative investment policies and levels of living expenses that are better suited for the retiree's risk tolerance and desired quality of life. The impact of these alternatives is easily evaluated by revising discretionary data and rerunning the optimization model. In the case of our typical couple, the limited use of deferred savings to meet total cash needs prior to age 73 leads to a recommendation by the couple's wealth manager to reallocate funds in this account to provide a greater average ROR in earlier years, even with higher expected volatility. A revised plan reallocates assets within this account for a slightly higher risk to an expected ROR of 8.0% for ages 66-72, returning to the moderate risk of 7.5% for ages 73-90. A rerun of the optimization

model (details are not presented) gives an expected final total account value of $1,733,188 which is 8.9% more than the initial plan.

4.6 Evaluation of the impact of the optimization model

In this section, we compare CW plans to tax-efficient plans for various levels of initial wealth allocated identically among the two account types: 10% in taxable savings, 90% in tax-deferred savings. Desired living expenses are not changed allowing evaluation of the impact of having more or less initial wealth relative to one's needs. Final wealth for CW and optimally determined tax-efficient withdrawal plans are compared for eleven levels of initial total wealth in Table 4 and Figure 2.

[Insert Table 4 and Figure 2 here]

Table 4 and Figure 2 show that the advantages of using an optimal tax-efficient policy over CW increase as the amount of initial wealth varies either above or below one million dollars - a level that could be considered in line with one's expense needs. As initial wealth decreases, the implication for wealth planning is that optimal tax-efficient policies can greatly improve the chances of retirees meeting their living expense goals during retirement. For retirees with relatively low savings, an optimal tax-efficient plan provides higher accumulated wealth than a CW plan and reduces the chance of exhausting wealth over the planning horizon. For retirees who are fortunate to have saved considerably more than needed for basic needs, optimal tax-efficient planning provides only a small advantage over a CW plan that draws from taxable savings first. This analysis raises questions as to how the percent improvement curve might change under different conditions, including, different percentages of initial wealth in taxable accounts, different RORs for these accounts, etc. These concerns will be addressed in future research.

4.7 Planning for occasional large withdrawals

Living expenses for retirees are not always as stable as suggested by our typical data. Occasional large withdrawals may be needed for car purchases, vacations, funding "retirement" businesses, etc. – see Van Harlow and Feinschreiber (2006). We now consider the scenario of new car purchases every five years at ages 70, 75 and 80 for $20,000, $25,000 and $30,000, respectively.

Optimal tax-efficient model results are shown in Table 5 and Figure 3.

[Insert Table 5 and Figure 3 here]

The total final wealth value is $1,327,000 - down from $1,592,000 without the car purchases (see Table 3). At age 70 the total cash needs of $108,400 includes the added cost of the car and is satisfied by Social Security ($27,000), taxable savings ($29,100) and tax-deferred savings ($52,300). Before age 72, more tax-deferred savings than needed are transferred to taxable savings to reduce the impact of being taxed at higher brackets in later years, especially when car purchases are made – even when the car is purchased at age 70, an additional $30,100 is transferred from tax-deferred to taxable savings. At age 73 a tax-deferred withdrawal of $86,566 above the RMD is added to taxable savings to cover the cost of the car at age 75, along with the RMD for that year and Social Security. Note that this excess withdrawal also reduces future RMDs. The $75,600 withdrawn from tax-deferred savings at age 79 includes about $4,000 more than the RMD that is deposited into taxable savings to help pay for the car at age 80. A CW plan (as described in Section 4.3) results in a final total remaining account value of $1,285,000 – the optimal tax-efficient result of $1,327,000 is 3.3% higher, or $42,000.

The implication for wealth planning is that occasional need for large withdrawals is expected, but planning for them in an optimal tax-efficient manner is complex. Using an optimal tax-

efficient model takes the guesswork out of this type of planning and automatically provides a withdrawal policy with significant financial benefits.

5. Summary and implications for future research

We describe an approach to retirement withdrawals that uses a mathematical optimization model. The use of this approach determines withdrawal amounts that maximize a specified measure of wealth and assures that future specified living expenses are met, taxes are paid and all relevant employer and/or government-imposed constraints are satisfied. We have analyzed typical scenarios that demonstrate some potential benefits that our approach can provide to retirees and their wealth managers.

Further research is needed to extend the methods of this paper to answer specific questions associated with retirement planning, including:

- What is the impact on retirement planning when income sources other than those considered in the prototype model are added? These sources could include tax-free savings (e.g., Roth IRAs, municipal bonds) and income from capital gains.
- Prior to retirement, what levels of savings are needed, and in which types of accounts, to satisfy desired retirement spending goals?
- At what age should Social Security be started?
- Is long-term care insurance needed? If so, how much? When should premiums be started?
- How can the retirement planning process developed in this paper be used in conjunction with ROR simulation to assess sustainability risk?

Appendix

The prototype optimization model specified below includes income from Social Security and two account types – taxable savings and tax-deferred savings. Annual withdrawals used for specified expenses are assumed to occur at the beginning of a year; excess withdrawals from tax-deferred savings used for purposes other than expenses (e.g., to satisfy federal RMDs) are assumed to occur at the end of a year.

A.1 Parameters

T: Planning horizon.

ACC: Set of accounts, that includes

AFT: Taxable savings.

DEF: Tax-deferred savings.

$ACCr_t$: Interest Rate/ROR for ACC at time t.

$P_{ACC}(a,b)$: ROR compound amount for ACC = $\prod_{j=a}^{b} (1+ ACCr_j)$.

AFTI: Initial taxable savings

DEFI: Initial tax-deferred savings

ss_t : Social Security payment in year t.

IDE_t: Itemized deductions and exemptions in year t.

SLE_t: Specified living expenses in year t.

U_q : Upper range of tax bracket q.

$TXRT_q$:Tax rate of tax bracket q.

FP_t: federal coefficient used in determining RMD in year t.

A.2 Variables

s_t: Taxable savings account withdrawal in year t.

d_t: Tax-deferred savings withdrawal used to meet living expenses in year t.

x_t: Amount transferred from tax-deferred to taxable savings in year t.

D_{qt}: Taxable income in tax bracket q in year t.

A.3 Objective Function

Max Final Accumulated Wealth =

$$\text{Max} \sum_{\forall \text{ ACC}} [\text{Final Wealth of ACC}]$$

Where:

- Final Wealth of ACC is a function of growth-adjusted (initial wealth less withdrawals plus any additions). The impact of initial wealth is constant and does not need to be included in the mathematical statement of the objective function.

Mathematically:

$$\text{Max} \quad -\left[\sum_{j=1}^{T} s_j * P_{AFT}(j,T) + \sum_{j=1}^{T} d_j * P_{DEF}(j,T)\right] + \sum_{j=1}^{T-1} x_j * [P_{AFT}(j+1,T) - P_{DEF}(j+1,T)] \quad \text{(A1)}$$

A.4 Constraints

A.4a An account withdrawal in year t must be $\leq$ total available at end of year (t-1).

For taxable savings:

$s_1 \leq \text{AFTI}$ for year 1 (A2)

$s_2 \leq \text{AFTI} * P_{AFT}(1,1) - s_1 * P_{AFT}(1,1) + x_1$ for year 2 (A3)

$$s_t \leq \text{AFTI} * P_{AFT}(1,t-1) - \sum_{j=1}^{t-1} s_j * P_{AFT}(j,t-1) + \sum_{j=1}^{t-2} x_j * P_{AFT}(j+1,t-1) + x_{t-1} \quad \text{for t=3… T} \quad \text{(A4)}$$

For tax-deferred savings:

$d_1 + x_1 \leq \text{DEFI}$ for year 1 (A5)

$d_2 + x_2 \leq \text{DEFI} * P_{DEF}(1,1) - d_1 * P_{DEF}(1,1) - x_1$ for year 2 (A6)

$$d_t + x_t \leq DEFI^* P_{DEF}(1,t-1) - \sum_{j=1}^{t-1} d_j^* P_{DEF}(j,t-1) - \sum_{j=1}^{t-2} x_j^* P_{DEF}(j+1,t-1) - x_{t-1} \quad \text{for } t=3\ldots T \qquad (A7)$$

A.4b Sum of taxable income by tax bracket = total taxable income

$$\sum_{j=1}^{6} D_{jt} - D_{0t} = TTI_t \qquad \text{for } t=1,\ldots,T \qquad (A8)$$

Where:

TTI_t = (Interest on taxable savings) + withdrawals from taxable accounts + Taxable Social Security less (itemized deductions + exemptions).

$$TTI_1 = (AFTr_1^* AFTI - AFTr_1^* s_1) + d_1 + x_1 + Tax\_ss_1 - IDE_1 \qquad \text{for year 1} \qquad (A9)$$

$$TTI_2 = AFTr_2^*[(AFTI - s_1)^*(1 + AFTr_1) + x_1 - s_2] + d_t + x_2 + Tax\_ss_2 - IDE_2 \qquad \text{for year 2} \qquad (A10)$$

$$TTI_t = AFTr_t^*[AFTI^* P_{AFT}(1,t-1) - \sum_{j=1}^{t-1} s_j^* P_{AFT}(j,t-1) - s_t + \sum_{j=1}^{t-2} x_j^* P_{AFT}(j+1,t-1) + x_{t-1}] + d_t + x_t +$$

$$Tax\_ss_t - IDE_t \qquad \text{for } t=3,\ldots T \qquad (A11)$$

NOTE: Tax_ss_t is determined approximately and the optimal withdrawal plan is determined based on this approximation. Taxable social security computed using optimal results and Federal worksheet procedures match to within one percent.

A.4c. D_q's constrained to taxable range in bracket q

$$D_{qt} \leq U_q - U_{q-1} \qquad q=1,\ldots, 5 \qquad \text{for } t=1,\ldots,T \qquad (A12)$$

NOTE: $U_0 = 0$.

A.4d Living expenses must be satisfied

Total Income – Taxes = Specified living expenses

$$\{s_t + d_t + ss_t\} - \sum_{j=1}^{6} TXRT_j^* D_{jt} = SLE_t \qquad \text{for } t=1,\ldots,T \qquad (A13)$$

A.4e Withdrawal for tax-deferred savings $\geq$ federal RMD

$$d_1 + x_1 \geq DEFI/FP_1 \qquad \text{if retiree is 70.5 in year 1} \qquad (A14)$$

$$d_2 + x_2 + \geq (DEFI^* (1 + DEFr_1) - d_1^*(1 + DEFr_1) - x_1)/FP_2 \qquad \text{for t=[Yr of 70.5=2]} \qquad (A15)$$

$$d_t + x_t \geq [DEFI * P_{DEF}(1,t-1) - \sum_{j=1}^{t-1} d_j * P_{DEF}(j,t-1) - \sum_{j=1}^{t-2} x_j * P_{DEF}(j+1,t-1) - x_{t-1}]/FP_t$$

for t=[Yr after 70.5=3], … T (A16)

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Figure 1 Graphical display of an optimal tax-efficient withdrawal plan

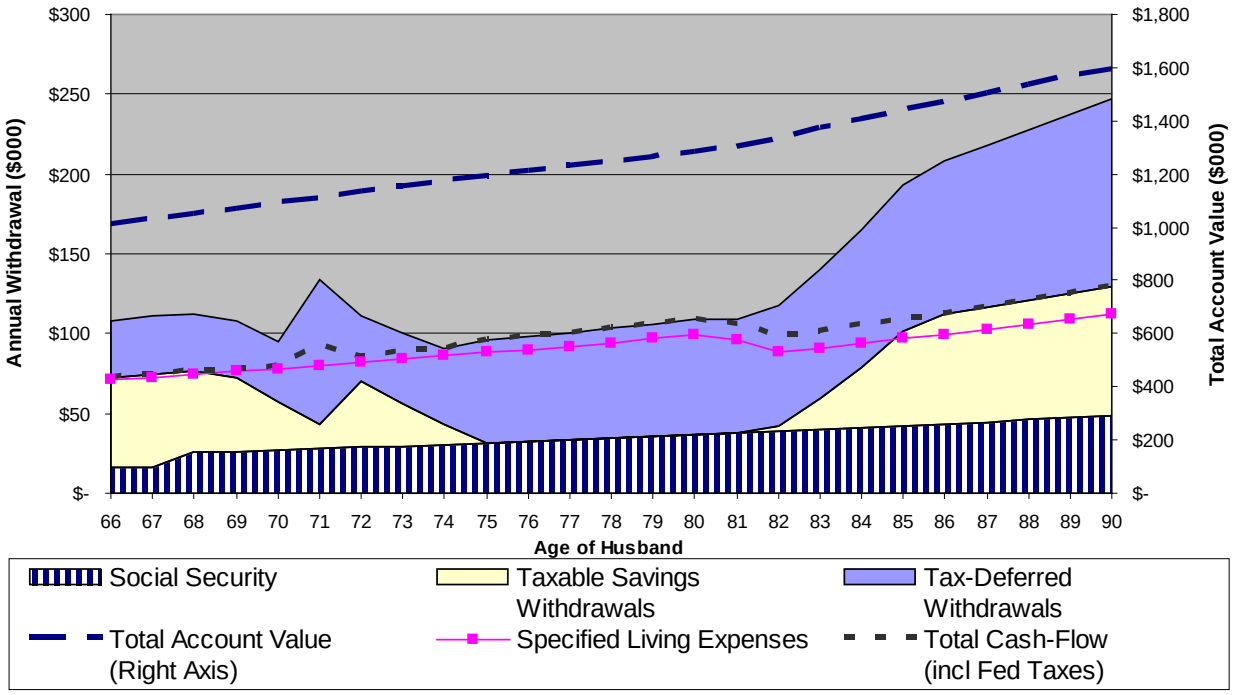
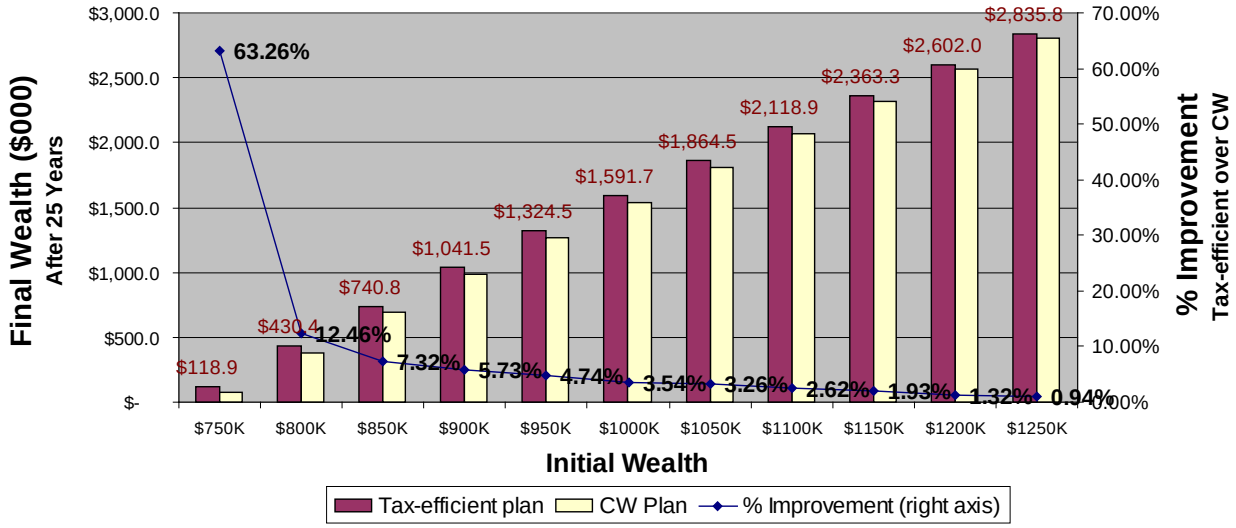


Figure 2 Graphical display comparing final wealth for optimal tax-efficient withdrawal plans versus CW plans that draw from taxable savings before any tax-deferred savings.



Note: Values above bars are for the tax-efficient plan.

Figure 3 Graphical display of an optimal tax-efficient withdrawal plan when cars are purchased every five years.

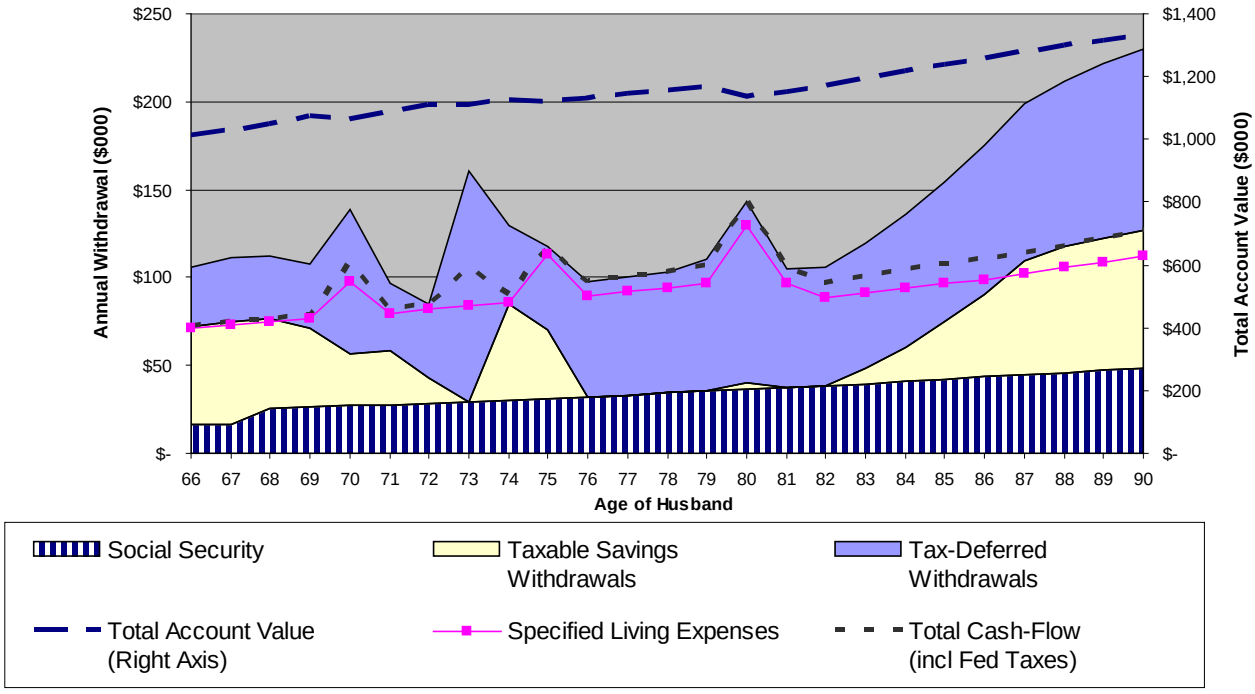


Table 1 Summary of current living expenses specified by a retired couple

Item	*Annual Amount*
Household	$40,800
Personal Care	$7,200
Transportation	$4,800
Leisure	$11,800
Miscellaneous	$4,800
Total	$69,400

Table 2 CW withdrawal plan (values in $000)

	Cash Needs			*Cash Sources/ Withdrawals*[2]			
Age of Husband	Specified Living Expenses	Approx. Federal Income Taxes	Total Cash-flow (incl Fed Taxes)	Social Security	Taxable Savings	Tax Deferred Savings	Total Remaining Account Value
66	$71.0	$ -	$71.0	$16.0	$55.0	$-	$ 1,015.0
67	$72.7	$ -	$72.7	$16.5	$47.5	$8.7	$ 1,030.7
68	$74.5	$ 6.4	$80.9	$25.5	$-	$55.4	$ 1,048.4
69	$76.3	$ 6.6	$82.9	$26.2	$-	$56.7	$ 1,066.0
70	$77.9	$ 6.8	$84.7	$27.0	$-	$57.7	$ 1,084.0
71	$79.8	$ 7.1	$86.9	$27.8	$-	$59.1	$ 1,101.8
72	$81.8	$ 7.3	$89.1	$28.7	$-	$60.4	$ 1,119.4
73	$83.9	$ 7.6	$91.5	$29.5	$-	$62.0	$ 1,136.8
74	$86.1	$ 7.8	$93.9	$30.4	$-	$63.5	$ 1,153.8
75	$88.3	$ 8.1	$96.4	$31.3	$-	$65.1	$ 1,170.4
76	$89.7	$ 8.2	$97.9	$32.3	$-	$65.6	$ 1,187.6
77	$92.0	$ 8.5	$100.5	$33.2	$-	$67.3	$ 1,204.4
78	$94.4	$ 8.9	$103.3	$34.2	$-	$69.1	$ 1,220.5
79	$96.9	$ 9.2	$106.1	$35.2	$-	$70.9	$ 1,235.8
80	$99.5	$ 9.5	$109.0	$36.3	$-	$72.7	$ 1,250.3
81[1]	$96.3	$ 9.2	$105.5	$37.4	$-	$69.8	$ 1,270.9
82	$88.0	$ 9.9	$97.9	$38.5	$1.8	$74.2	$ 1,302.3
83	$90.9	$ 10.5	$101.4	$39.7	$16.6	$78.9	$ 1,333.7
84	$93.8	$ 11.2	$105.0	$40.9	$33.8	$83.9	$ 1,364.8
85	$96.8	$ 11.8	$108.6	$42.1	$53.5	$88.6	$ 1,395.6
86	$99.0	$ 12.6	$111.6	$43.3	$68.3	$93.6	$ 1,426.8
87	$102.1	$ 13.5	$115.6	$44.6	$71.0	$98.9	$ 1,456.9
88	$105.5	$ 14.4	$119.9	$46.0	$73.9	$104.4	$ 1,485.5
89	$108.9	$ 15.5	$124.4	$47.4	$77.0	$110.1	$ 1,512.4
90	$112.4	$ 16.4	$128.8	$48.7	$80.1	$114.9	**$ 1,537.4**

NOTES:
[1] The drop in living expenses is due to the termination of the mortgage.
[2] When the cash source total exceeds cash needs, the excess is withdrawn at year end and deposited into taxable savings. This excess may be due to the RMD of tax-deferred savings.

Table 3 Optimal tax-efficient withdrawal plan (values in $000)

Age of Husband	*Cash Needs*			*Cash Sources/ Withdrawals*[2]			Total Remaining Account Value
	Specified Living Expenses	Approx. Federal Income Taxes	Total Cash-flow (incl Fed Taxes)	Social Security	Taxable Savings	Tax Deferred Savings	
66	$71.0	$1.7	$72.7	$16.0	$56.7	$35.0	$ 1,013.2
67	$72.7	$1.7	$74.4	$16.5	$57.9	$36.4	$ 1,026.5
68	$74.5	$1.8	$76.3	$25.5	$50.8	$35.5	$ 1,048.7
69	$76.3	$1.8	$78.1	$26.2	$45.6	$36.3	$ 1,071.5
70	$77.9	$2.3	$80.2	$27.0	$30.0	$38.0	$ 1,094.8
71	$79.8	$12.5	$92.3	$27.8	$14.9	$90.9	$ 1,107.6
72	$81.8	$3.3	$85.1	$28.7	$41.3	$41.7	$ 1,130.0
73	$83.9	$4.1	$88.0	$29.5	$26.5	$44.7	$ 1,151.9
74	$86.1	$4.9	$91.0	$30.4	$12.7	$47.9	$ 1,173.1
75	$88.3	$8.1	$96.4	$31.3	$0.0	$65.1	$ 1,191.1
76	$89.7	$8.2	$97.9	$32.3	$0.0	$65.6	$ 1,209.9
77	$92.0	$8.5	$100.5	$33.2	$-	$67.3	$ 1,228.3
78	$94.4	$8.9	$103.3	$34.2	$0.0	$69.1	$ 1,246.2
79	$96.9	$9.2	$106.1	$35.2	$-	$70.9	$ 1,263.5
80	$99.5	$9.5	$109.0	$36.3	$-	$72.7	$ 1,280.1
81[1]	$96.3	$9.4	$105.7	$37.4	$-	$71.5	$ 1,302.6
82	$88.0	$10.1	$98.1	$38.5	$3.2	$76.0	$ 1,336.2
83	$90.9	$10.8	$101.7	$39.7	$19.5	$80.8	$ 1,369.8
84	$93.8	$11.5	$105.3	$40.9	$38.3	$85.9	$ 1,403.2
85	$96.8	$12.2	$109.0	$42.1	$59.8	$90.8	$ 1,436.6
86	$99.0	$13.0	$112.0	$43.3	$68.7	$96.0	$ 1,470.3
87	$102.1	$13.9	$116.1	$44.6	$71.5	$101.4	$ 1,503.0
88	$105.5	$14.9	$120.4	$46.0	$74.4	$107.0	$ 1,534.3
89	$108.9	$16.0	$124.9	$47.4	$77.5	$112.8	$ 1,563.9
90	$112.4	$17.0	$129.4	$48.7	$80.7	$117.8	**$ 1,591.7**

NOTES:
[1] The drop in living expenses is due to the termination of the mortgage.
[2 When the cash source total exceeds cash needs, the excess is withdrawn at year end and deposited into taxable savings. This excess may be due to the RMD of tax-deferred savings and/or tax bracket management.

Table 4 Comparison of final wealth for optimal tax-efficient withdrawal plans versus CW plans that draw from taxable savings before tax-deferred savings (values in $000)

	Final Wealth		Improvement: Tax-efficient Over CW	
Initial Wealth	Tax-efficient	CW	Amount	%
$750	$118.9	$72.9	$46.1	63.26%
$800	$430.4	$382.7	$47.7	12.46%
$850	$740.8	$690.3	$50.5	7.32%
$900	$1,041.5	$985.1	$56.5	5.73%
$950	$1,324.5	$1,264.5	$60.0	4.74%
$1000	$1,591.7	$1,537.4	$54.4	3.54%
$1050	$1,864.5	$1,805.7	$58.8	3.26%
$1100	$2,118.9	$2,064.8	$54.2	2.62%
$1150	$2,363.3	$2,318.6	$44.7	1.93%
$1200	$2,602.0	$2,568.1	$33.8	1.32%
$1250	$2,835.8	$2,809.5	$26.3	0.94%

Table 5 Optimal tax-efficient withdrawal plan when cars are purchased every five years – at ages 70, 75, 80 (values in $000).

Age of Husband	*Cash Needs*			*Cash Sources/ Withdrawals*[2]			
	Specified Living Expenses	Approx. Federal Income Taxes	Total Cash-flow (incl Fed Taxes)	Social Security	Taxable Savings	Tax Deferred Savings	Total Remaining Account Value
66	$71.0	$ 1.5	$72.5	$16.0	$56.5	$33.8	$ 1,013.4
67	$72.7	$ 1.7	$74.4	$16.5	$57.9	$36.5	$ 1,026.7
68	$74.5	$ 1.8	$76.3	$25.5	$50.8	$35.5	$ 1,049.0
69	$76.3	$ 1.8	$78.1	$26.2	$44.7	$36.3	$ 1,071.8
70	***$97.9***	$ 10.5	$108.4	$27.0	$29.1	$82.4	$ 1,064.7
71	$79.8	$ 2.5	$82.3	$27.8	$30.1	$39.0	$ 1,086.0
72	$81.8	$ 3.3	$85.1	$28.7	$14.6	$41.8	$ 1,106.7
73	$83.9	$ 22.3	$106.2	$29.5	$0.0	$131.4	$ 1,107.3
74	$86.1	$ 3.9	$90.0	$30.4	$54.7	$44.2	$ 1,126.3
75	***$113.3***	$ 4.8	$118.1	$31.3	$39.3	$47.5	$ 1,117.4
76	$89.6	$ 8.2	$97.8	$32.3	$-	$65.5	$ 1,130.7
77	$92.0	$ 8.5	$100.5	$33.2	$-	$67.3	$ 1,143.2
78	$94.4	$ 8.9	$103.3	$34.2	$-	$69.1	$ 1,154.7
79	$96.9	$ 9.9	$106.8	$35.2	$-	$75.6	$ 1,164.4
80	***$129.5***	$ 14.1	$143.6	$36.3	$3.9	$103.4	$ 1,136.3
81[1]	$96.3	$ 8.9	$105.2	$37.4	$0.0	$67.8	$ 1,148.7
82	$88.1	$ 8.8	$96.9	$38.5	$-	$67.2	$ 1,172.1
83	$90.9	$ 9.4	$100.3	$39.7	$8.8	$71.4	$ 1,194.9
84	$93.8	$ 10.0	$103.8	$40.9	$19.6	$75.8	$ 1,216.8
85	$96.8	$ 10.5	$107.3	$42.1	$32.5	$80.0	$ 1,237.9
86	$99.0	$ 11.1	$110.1	$43.3	$47.3	$84.4	$ 1,259.0
87	$102.2	$ 11.8	$114.0	$44.6	$65.0	$89.1	$ 1,279.0
88	$105.5	$ 12.5	$118.0	$46.0	$72.0	$94.0	$ 1,297.3
89	$108.9	$ 13.4	$122.3	$47.4	$74.9	$99.1	$ 1,313.4
90	$112.4	$ 14.1	$126.6	$48.7	$77.9	$103.5	**$ 1,327.1**

NOTES:
[1] The drop in living expenses is due to the termination of the mortgage.
[2] When the cash source total exceeds cash needs, the excess is withdrawn at year end and deposited into taxable savings. This excess may be due to the RMD of tax-deferred savings and/or tax bracket management.